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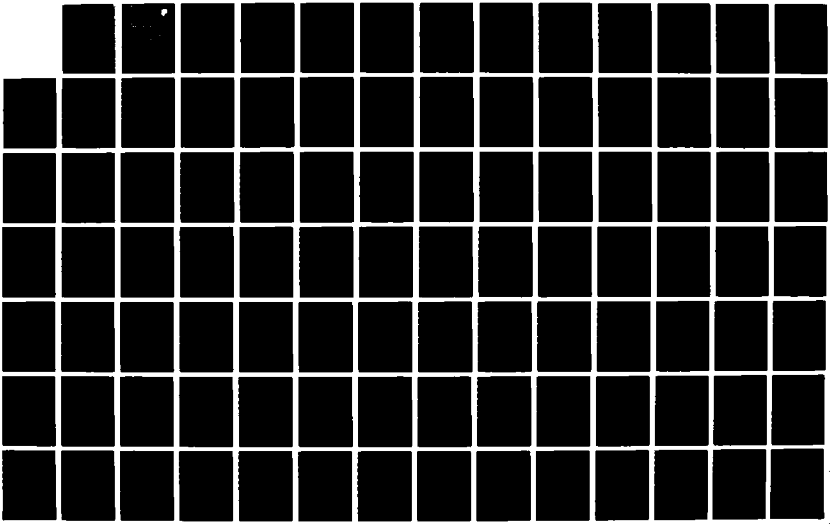

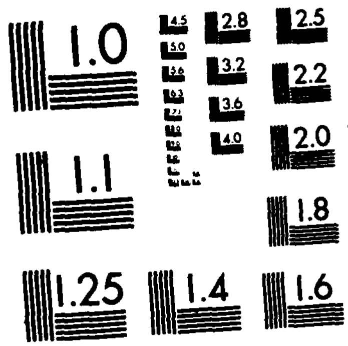

MICROCOPY RESOLUTION TEST CHART
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INTRODUCTION TO RELIABILITY DEMONSTRATION TESTING OF NONELECTRONIC COMPONENTS

John P. LaFollette, LTC, USAFR

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
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This report is intended as an instructional manual in demonstration testing of nonelectronic components. Primary emphasis is upon truncated tests. Starting from a basic binomial model, the trade-offs between sample size, consumer risk, rejection number and the expected probability of failure are explored. The Weibull distribution is a flexible model for representing mechanical failure modes where reliability degrades with time (a hazard rate that increases with time). The time of test termination follows from the appropriate failure model and the desired reliability. Techniques are presented for analyzing truncated failure data to determine an appropriate failure model. The problems and advantages of using current military test specifications, designed for an exponential failure model, when testing mechanical components, are discussed in detail. The Probability Ratio Sequential Test (PRST) is discussed as an alternative to the truncated test. A variety of PRSTs for different failure models are presented. The Weibull PRST and the exponential PRST are compared.

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I INTRODUCTION

The RADC Reliability Guide for Nonelectronic Component testing is written to provide guidance to the Reliability Engineer and others interested in testing mechanical components. Mechanical components, different from electronic, frequently fail from wear so that the probability of the component failing is likely to increase with time whereas electronic components frequently are no more likely to fail after two hours of operation than following the first hour of operation. It is important that the mathematical models underlying a particular test plan properly describe the true failure history. Otherwise, an incorrect decision may be made to accept or to reject the sample. As will be shown, for mechanical components, usage of an exponential test specification, commonly used for electronic components, may well result in the error of rejecting good components.

Determining the correct statistical model to describe the components failure history is only one of several decisions that this manual is written to clarify. Also to be considered is selection of the sample size, test time, and type of reliability demonstration test to be used.

The procedure followed in this handbook assumes no knowledge as to the nature of how the components fail over time except what knowledge may be gleaned from the test data. For purposes of making quick, economical decisions, the handbook focuses upon time terminated tests and the Wald probability ratio sequential test.

Failure distributions addressed in the handbook are the binomial, the exponential, and the Weibull. Reference is made to the more commonly used normal distribution. It is not the distribution of choice for reasons that will be discussed later.

Where applicable, reference is made to the standard Department of Defense test specifications. It is hoped that this handbook will clarify some of the concepts used in designing the specifications.

II What Do We Mean - Reliability

Reliability is a proportional concept, a fraction. It is the answer to the question: "What proportion of all components, built to specified design, will operate successfully under specified conditions?" It is important to note the need for specified designs and specified conditions. In a sampling plan, both must be representative of production components operated in a realistic operating environment. Testing a hand picked, "salted", sample to less than stringent operating conditions is a sure prescription to disaster in the field.

One-shot devices provide a good introduction to reliability demonstration testing and the concepts of confidence and sample size. Testing of a go no-go characteristic such as successful firing of an explosive squib or the maximum length of some critical dimension is known as a "test by attribute".

MIL-STD-105D, "Sampling Procedures and Tables for Inspection by Attributes,"provides a standardized approach to this type of testing and is widely specified in Air Force contracts. Testing by attribute is based upon the binomial expression (sometimes called the Bernoulli formula) as follows:

$$P(S=r|n,p) = \frac{n!}{r!(n-r)!} \cdot p^r(1-p)^{n-r}$$

This expression states that the probability of exactly r successes given a sample of n items, from a population in which the probability of success is p, equals:

$$\frac{n!}{r!(n-r)!} \cdot p^r(1-p)^{n-r}$$

The general case is that we do not know the population reliability, p. By test, however, an estimate of the reliability can be made by simply dividing the number of items passing the test by the total number tested. The problem is that, while the success ratio may be the most likely estimate of reliability, the true reliability could be quite different. Suppose that five items were tested and one failed. Figure 1 shows the probability of various possible numbers of failures and successes for a reliability of .5 (fifty percent will fail if many items are tested).

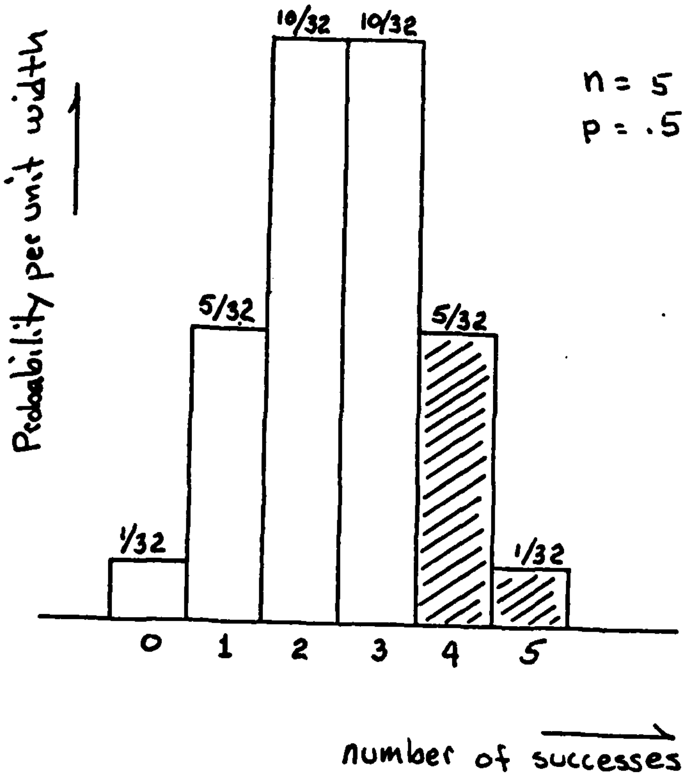


FIGURE 1
Example of Testing Five Items With One Failure

Five out of every thirty two five-item samples that we may test are expected to have one and only one failure, if the true reliability is as low as 0.5. Six out of every thirty two five-item samples are expected to exhibit at most one failure (0 or 1). Having performed a test with four observed successes, we know that, if the true reliability is as low as 0.5, there are only six chances in thirty two of obtaining the test results. A conclusion that the true reliability is at least 0.5 thus has a limited risk of being wrong. Subtracting this risk from one provides a measure of confidence in the conclusion.

Summing the binomial expression gives:

$$P(S \leq r \mid n, p) = \sum_{S=0}^{r} \frac{n!}{S!(n-S)!} \cdot p^{S}(1-p)^{n-S}$$

$$P(S \geq r \mid n, p) = \sum_{S=r}^{n} \frac{n!}{S!(n-S)!} \cdot p^{S}(1-p)^{n-S}$$

The summation expressions are used to define a range within which p must exist if the expressions are to be satisfied. The defined range is called a confidence interval. "p" may increase to a defined maximum, called the upper confidence level (UCL), or p may decrease to a defined minimum, called the lower confidence level (LCL).

Thus, the quantities

$$C_U = \sum_{S=0}^{r} \frac{n!}{S!(n-S)!} \cdot p^{S}(1-p)^{n-S}$$

and

$$C_L = \sum \frac{n!}{S!(n-S)!} \cdot p^{S}(1-p)^{n-S}$$

determine upper and lower limits for p given C_U and C_L.
Where:

s = number of successes which may expand up to r, the number of observed success

n = number of items tested

p = probability of success

C_U and C_L are cumulative probabilities chosen by the test engineer as a measure of the risk that the true probability of success (reliability) may exist outside the UCL or LCL.

The use of the terms "upper" and "lower" in the confidence interval concept refer to the random variable being estimated, p in the above case. Thus a statement that one is 90 percent confident that the mean height of nine year old males is between 4 ft. 6 in. and 5 ft. even means that, based upon analysis of some earlier sample, the true mean has a low probability (0.05) of exceeding these extremes. This is known as a two tailed confidence limit. C_U and C_L equal 0.05, or one half of the difference between the confidence range and unity. The UCL is 5 feet and the LCL is four feet six inches. A statement that one is 90 percent confident that the mean height of nine year old males is at most 4 feet 11 inches is a single tail confidence limit in which C_U equals 0.10 and the UCL is 4 feet 11 inches. In the latter case, the UCL of 4 feet 11 inches corresponds to the UCL of a two tailed statement that one is 80 percent confident that the mean height is between 4 feet 7 inches and 4 feet 11 inches. The range between the UCL and LCL is less at the lower confidence due to accepting a larger risk of the conclusion being wrong.

Unfortunately, the binomial formulas are not easily solved without a computer. Several sources of tabulated solutions exist, some of which are included in the bibliography. These tables present the cumulative binomial distribution in various manners. Reference 7, for instance tabulates:

$$P(S \leq r \mid n, p)$$

Reference 4 tabulates:

$$P(S > r \mid n, P)$$

Most binomial tables have columns where p varies up to p = .5. These tables are used with p equal to the probability of failure and r equal to the number of failures. What has been written about upper and lower confidence limits is consistent with any definition of p in the sense that the UCL is the higher value of p and LCL is the lower value of p. Of course, if p is defined as the probability of failure, its LCL corresponds to the UCL for the corresponding probability of success (reliability).

For future discussion define reliability as $R = P_R$ = Probability of success and P_F = Probability of failure.

$P_R + P_F = 1$, there being no other alternatives.

Shown below are selected columns from a table of the cumulative binomial distribution excerpted from Reference 7. As an example, let's use these excerpts to determine the LCL and UCL for a test which resulted in 40 successes following a test of 50 items. Since Reference 7 only tabulates p from 0 to 0.5, we know immediately that confidence limits for P_F must be determined and later subtracted from one to give limits for P_R. For example, to determine a two sided 90 percent confidence interval, let x in the tables equal the number of failures (10 in this case). Looking through the

tables, we notice that the UCL for p exists between p = .31 and p = .32 (Table 1) for x = 10 and a cumulative probability of .05. In order to find the LCL for p, we have to make use of the complement of C_L which in this case is 1 - .05 = .95. The LCL for p exists between p = .12 and p = .13 (Table 2) for x = 10 and a cumulative probability of .95. The tables present solutions to the equations:

$$C_U = .05 \doteq \sum_{f=0}^{10} \frac{50!}{f!(50-f)!} \cdot (P_F)^f (P_R)^{50-f}$$

and:

$$1 - C_L = .95 \leq \sum_{f=0}^{10} \frac{50!}{f!(50-f)!} \cdot (P_F)^f (P_R)^{50-f}$$

where:

P_F = Probability of Failure
P_R = $1 - P_F$ (Reliability)
f = number of failures (x as used in tables)

Note that we are looking for confidence limits on P_F, not the reliability. Interpolating for a more precise solution provides:

$$\text{LCL (for } P_F) = .12 + .01 \times \left[\frac{.967501 - .95}{.967501 - .946505}\right] = .1283$$

$$\text{UCL (for } P_F) = .31 + .01 \times \left[\frac{.059137 - .05}{.059137 - .043663}\right] = .3159$$

To determine a 90 percent confidence limit for P_R, subtract from one to give:

UCL (for P_R) = 1 - .1283 = .8717
LCL (for P_R) = 1 - .3159 = .6841

Notice that the interval brackets the ratio of successes to total number tested.

N = 50

p = .31

x	Individual Term	Cumulative (x or less)
.		
.		
9	.016366	.028989
10	.030147	.059137
11	.049252	.108389
.		

p = .32

x	Individual Term	Cumulative (x or less)
.		
.		
9	.011970	.043633
10	.023095	.043663
11	.039521	.083185
.		

Table 1
Cumulative Binomial Probability

N = 50

p = .12

x	Individual Term	Cumulative (x or less)
.		
.		
9	.068439	.929238
10	.038264	.967501
11	.018974	.986475
.		

p = .13

x	Individual Term	Cumulative (x or less)
.		
.		
9	.088036	.892570
10	.053935	.946505
11	.029306	.975811
.		

Table 2
Cumulative Binomial Probability

Suppose that 100 items had been tested instead of 50 items with 80 items passing. Review of the binomial tables (not presented here) shows that, even though the point reliability estimate, R = .80 = 80/100 is the same, the 90 percent confidence limits narrow to:

UCL = .8533

LCL = .7244

Increasing the sample size narrows the range within which the engineer is relatively confident of his reliability estimate. This narrowing, however, was achieved at double the cost, assuming that costs are proportional to the size of the sample tested.

Tests which terminate at some pre-determined time are known as time truncated tests. Since these tests are basically pass/fail tests, they are essentially binomial tests similar to testing a lot of squids. As discussed in this chapter, reliability estimates vary depending upon sample size and the risk which the user is willing to accept should the estimate be wrong. Later chapters will explore the nature of the trade-offs necessary in choosing a given demonstration test, including considerations of how items fail with time.

III Demonstration Testing

Usually, an engineer is more interested in determining that equipment meets or exceeds a contractually specified reliability requirement as opposed to estimating the reliability within a confidence interval. The approach is similar to determining the upper confidence level. The problem is to solve for r in the formula:

$$1-\alpha = \beta = \sum_{f=0}^{r} \frac{n!}{f!(n-f)!} (P_F)^f \cdot (P_R)^{n-f}$$

In solving for r, P_F (the probability of failure) n (the sample size) and α (the required confidence level) is specified. As an example of using binomial tables for the above problem consider the following tests. A sample of 100 items is tested to insure a reliability of no less than .90 ($P_F \geqq .1$) at a 90 percent confidence level ($\alpha = .9$). The selection of r, for which the probability of rejection exceeds 0.1, is $r \geq 6$. This is obtained from the following table by noting how large r can become until the tabulation exceeds .10 under the p = .10 column. Notice that for a sample of 50 items, failure of two ($r \geq 2$) or more items results in rejection. For a given confidence level, reducing the sample size requires a reduction in the rejection level.

N = 100
p = .10

x	Individual Term	Cumulative (x or less)
.		
.		
5	.033866	.057577
6	.059579	.117156
7	.088895	.206051
.		

N = 50
p =.10

x	Individual Term	Cumulative (x or less)
.		
.		
1	.028632	.033786
2	.077943	.111729
3	.138565	.250294
.		

Table 3
Cumulative Binomial Probability

In this example, r is increased until the right hand side of the above equation exceeds .1 which determines the rejection number, r. If a sample of n is then tested, the sample is rejected if r or more items fail. Rejection means that the test engineer is no longer α percent confident that the reliability equals or exceeds his specification level. α, as used here, is within the same context as producer's risk used in statistical quality control. It is an expression of the probability of rejecting a lot which in fact has an acceptable reliability. In the theory of statistical inference, this type of risk - rejection of a hypothesis which is true - is known as Type I error. The complement of the producer's risk, the consumer's risk is designated by the symbol β. For the above equation $\beta = 1 - \alpha$. The consumer's risk refers to the probability of accepting a lot whose reliability is in fact less than acceptable. In the theory of statistical inference, this type of risk - acceptance of a hypothesis which is false - is known as a Type II error.

Let us review again the concepts of confidence, consumer risk, and producer risk. Suppose that a test is designed which results in the statement that, as a result of the test, we are 90 percent confident that the reliability of a component is at least .98. We are accepting a 10 percent consumer's risk (100-90) that the component may actually not be so reliable. In agreeing to the test, the producer has accepted a large (90 percent) risk that his equipment may fail the test when in fact its reliability is adequate. Producers generally are not prepared to accept such high risk. This is why most test plans are designed to accept, with high consumer's risk (low confidence) highly reliable components while rejecting with low consumer's risk (high confidence) a considerably less reliable component.

As determined previously, testing 50 items and obtaining no more than one failure would be the lot acceptance criteria to demonstrate a 0.9 reliability with less than 10 percent consumer's risk. This is a difficult test to pass. For guidance in lot acceptance using the binomial distribu-

TABLE II-A—Single sampling plans for normal inspection (Master table)

(See 9.4 and 9.5)

Sample size code letter	Sample size	Acceptable Quality Levels (normal inspection)																									
		0.010	0.015	0.025	0.040	0.065	0.10	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0	6.5	10	15	25	40	65	100	150	250	400	650	1000
		Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re	Ac Re
A	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↓	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45
C	5	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑
D	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑
E	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	30 31	44 45	↑	↑	↑
F	20	↓	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑
G	32	↓	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑
H	50	↓	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑
J	80	↓	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑
K	125	↓	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
L	200	↓	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
M	315	↓	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
N	500	↓	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
P	800	↓	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
Q	1250	0 1	↑	↓	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R	2000	↑	↑	1 2	2 3	3 4	5 6	7 8	10 11	14 15	21 22	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑

↓ = Use first sampling plan below arrow. If sample size equals, or exceeds, lot or batch size, do 100 percent inspection.
↑ = Use first sampling plan above arrow.
Ac = Acceptance number.
Re = Rejection number.

Table - 4
Sampling Plan from MIL-STD-105D

SINGLE
NORMAL

tion, the Air Force uses MIL-STD-105D. MIL-STD-105D recommends sample code H for a test of fifty items to demonstrate an Average Quality Level (AQL) of ten percent defective. Sample code H provides for acceptance if no more than ten items fail, as shown in the attached Table II-A, excerpted from MIL-STD-105D.

Figure 2, also from MIL-STD-105D, shows a plot of various Operating Characteristic curves for sampling plans consistent with sample code H. An Operating Characteristic curve is simply a graph showing how the probability of passing the test varies with different percentage defective for the lot from which the sample is taken. It is thus a plot of consumer's risk versus probability of failure. Figure 2 tells the user of MIL-STD-105D that when he specifies sample code H for a desired AQL of 10 percent that, if the true quality is 10 percent defective, almost 100 percent of the lots tested will pass. If the true quality level was 20 percent defective, approximately 60 percent of all lots tested will pass. Use of MIL-STD-105D can be confusing since the standard is designed for multiple defects as well as a single defect per item. It is not the standard of preference for reliability analysis.

TABLE X-H—Tables for sample size code letter: H

CHART H - OPERATING CHARACTERISTIC CURVES FOR SINGLE SAMPLING PLANS

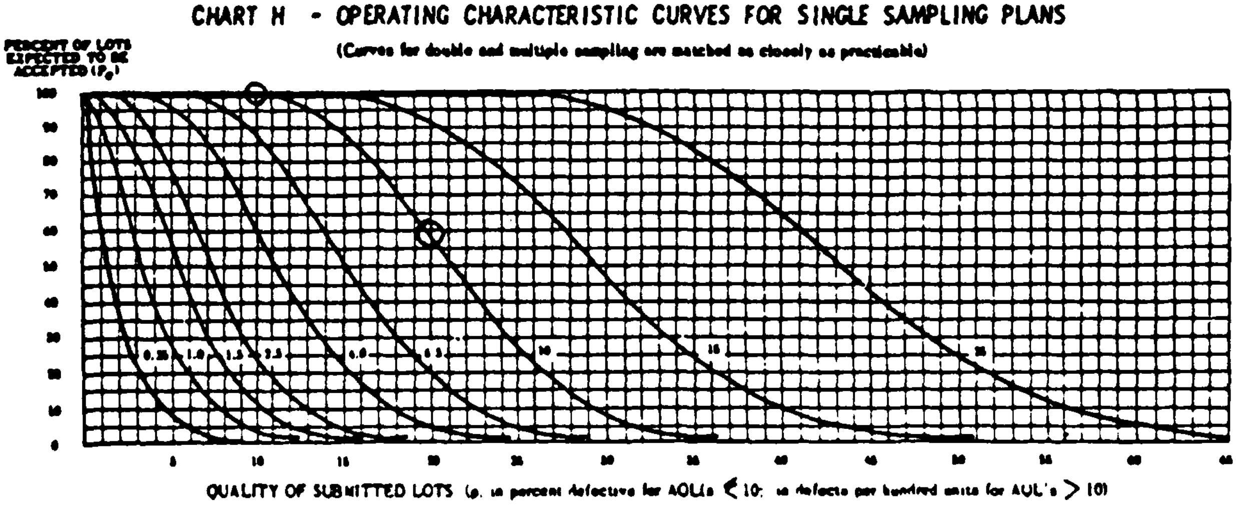


Figure 2. O-C Curves, MIL-STD-105D

IV AN APPROXIMATION TO THE BINOMIAL TABLES

Regardless of how components fail over time, any truncated test will result in a single outcome wherein the engineer will know the number of items tested, how long each item is tested, and the number of items which performed successfully. A binomial table can be used to determine the upper, or lower, confidence limit for the reliability at the time of test termination. Another method, which provides a useful approximation and which uses a much simpler table, is the chi-square (χ^2) distribution. Appendix 1 presents a table of chi-square values for different degrees of freedom (ν) and confidence levels (p). Let us look at how this table may be used in lieu of a binomial table to estimate reliability.

The formula for estimating the lower confidence level for the mean time to failure, Θ, using data from a failure truncated test, when the exponential failure distribution is known to exist, is as follows:

$$\Theta = \frac{2T}{{}^{2}_{2r+2,P}}$$

where

$T = \sum t$, sum of total sample test time

r = number of failures

P = desired consumer confidence

To use Appendix 1, the confidence level is read across the top of the table as "p" and the degrees of freedom are read down the margin ($\nu = 2r + 2$ in the above formula).

If the exponential distribution is defined by its reliability function

$$R = e^{-\frac{t}{\Theta}}$$

and all failures are assumed to occur at t = 1 then 2T becomes 2n and

$$R_T = e^{-\frac{\chi^2_{2r+2,P}}{2n}}$$

where

n = number of items tested

r = number of failures

P = confidence

Comparing with the earlier examples of a ninety percent upper and lower confidence limits on a sample of fifty items where ten fail and on a sample of one hundred items where twenty fail, the following table is excerpted from Appendix 1.

	P	
ν	.05	.95
22	12.3	33.9
42	28.1	58.1

NOTE: ν = 2r+2

Substituting into the above formula gives:

For a test of fifty items

$$UCL = e - \frac{12.3}{100} = .8843$$

$$LCL = e - \frac{33.9}{100} = .7125$$

For a test of one hundred items

$$UCL = e - \frac{28.1}{200} = .8689$$

$$LCL = e - \frac{58.1}{200} = .7479$$

This is the same computation that was performed earlier by method of interpolation of the binomial tables. This provided:

For a test of fifty items

UCL = .8717

LCL = .6841

For a test of one hundred items

UCL = .8533

LCL = .7244

The reader may judge for himself whether use of the chi-square distribution in the above manner is sufficiently accurate. The engineer is using a distribution applicable to exponential failures over time and assuming that all failures occur at test termination. The technique provides a relatively accurate and easily obtainable approximation.

The binomial distribution can be more accurately approximated with the Poisson or with the normal distribution. Each has its advantages depending upon the range of p. While the binomial distribution is the basis for choosing accept-reject criteria for truncated tests, these criteria have already been specified in applicable test specifications and there is little need to use the binomial distribution. Accordingly, no effort will be made to discuss other approximation methods. Refer to standard statistical textbooks for further clarification on developing binomial tables.

V The Sample Size VS Consumer Risk

There is nothing magic about any particular sample size. The engineer is free to select any sample size and accept-reject level consistent with his desired confidence. We might also say, consistent with his budget. Testing costs money. For this reason, it is important to understand how the variables interrelate. Figure 3 shows how consumer's risk increases as sample size decreases for a given desired reliability level, R = .9, and rejection number, r = 4.

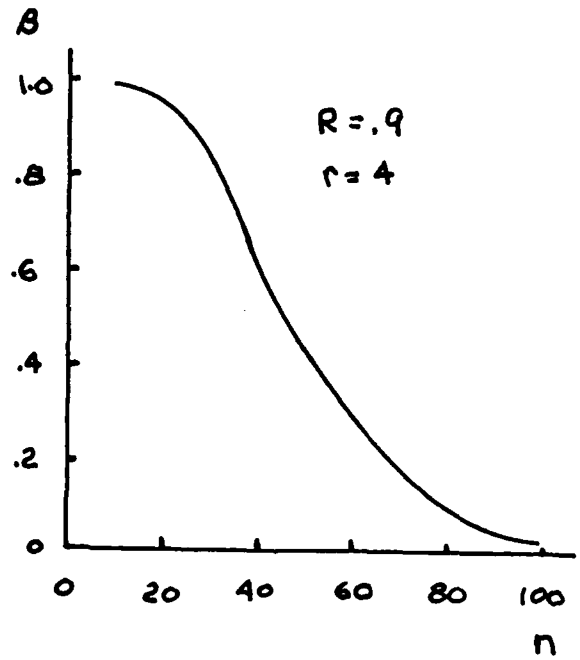


Figure 3
Consumer Risk vs Sample Size

If the shape of Figure 3 looks familiar, it is for a good reason. The curve is simply another way of presenting the familiar "operating characteristic curve" for a single sampling plan. This curve is commonly portrayed in the Military Specifications and numerous books on statistical quality control.

VI The Sample Size VS Reliability Trade-Off

Selecting a particular test involves the selection of several interrelated parameters which must satisfy the binomial equation. The equation is sufficiently complex in that it is difficult to visualize the relations pertinent to the decision. To clarify these relationships, this and following sections will use data extracted from a binomial table and presented in a graphical form. The following Table 5 and two graphs (Figures 4 and 5) present solutions to the cumulative binomial equation for four failures in testing a sample of n items to determine an upper 90 percent confidence level (β, probability of accepting a bad lot, equals 0.10) for the probability of failure, P_F.

n	P_F	$n \cdot P_F$
100	.07835	7.835
90	.08686	7.817
80	.09744	7.795
70	.11096	7.767
60	.12883	7.730
50	.15355	7.677
40	.18998	7.599
30	.24899	7.470
20	.36066	7.213

$r = 4$
$\beta = .1$

Table 5

PROBABILITY OF FAILURE VS SAMPLE SIZE

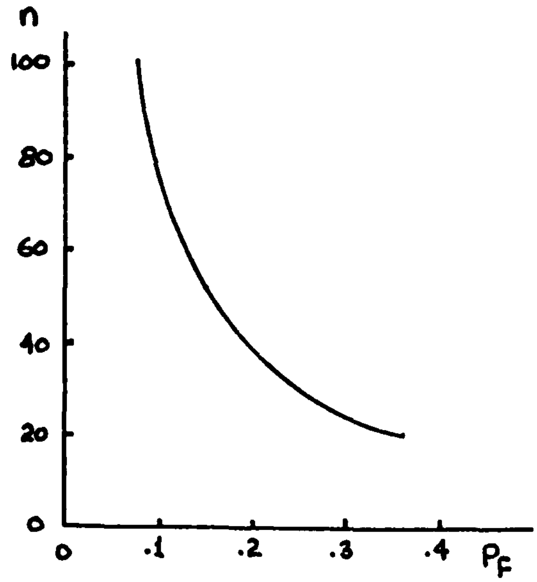


FIGURE 4

PROBABILITY OF FAILURE VS SAMPLE SIZE

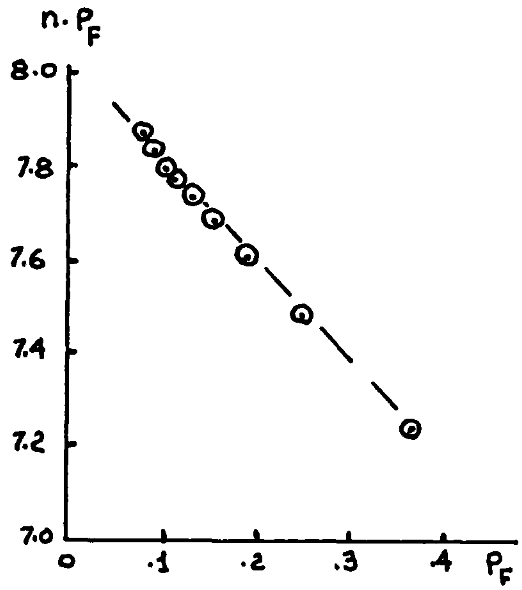


FIGURE 5

P_F VS N - LINEAR RELATIONSHIP

The slope of the near linear relationship shown in figure 5 is exaggerated due to the choice of scale. Notice that the product of the probability of failure times the sample size is nearly constant over the range analyzed. This means that, for a given acceptable number of failures and confidence level, the upper confidence level for P_F decreases directly as the sample size increases. Remember that reliability is the complement of the probability of failure $(1-P_F)$. Thus, in order to maintain confidence in a higher reliability, the sample size must increase with no additional failures. This trade-off becomes important in selecting a test termination time to be discussed in a later section.

VII The Sample Size VS Rejection Number Trade-Off

The test engineer has a choice not only of confidence level (or β) but also of the rejection number, r. Since some of the following graphs and tables are presented in a slightly different manner, let us pause to discuss nomenclature. Heretofore, we have used the symbol r to denote the termination number, meaning the minimum number of failures incurred in testing n items which, if encountered, will result in rejecting the lot from which the sample was obtained. Sometimes an acceptance number ($A_c=r-1$) is used in lieu of a rejection number. Following is a table of how the various military specifications specify rejection or, as the case may be, acceptance.

MIL-STD-105D

A_c = acceptance number

R_e = rejection number

DOD Handbook H-108

r = Termination number, the number of failures upon which a failure terminated test is to be terminated

P_0 = Acceptable proportion of a lot failing before a specified time

MIL-STD-781C

r_0 = the critical (reject) number of failures

MIL-STD-781C contains two types of test plans. The most widely used test is Wald's Probability Ratio Sequential Test (PRST), to be discussed later. The second test plan is a time terminated test in which the lot is rejected if r_0 failures have occurred. Both DOD Handbook H-108 and MIL-STD-781C are for testing components which fail exponentially.

Using the definitions of MIL-STD-105D, the data shown in Table 6 were extracted from a binomial table. When this data is plotted, as in figures 6 and 7, the near linear relationship of the variables is apparent. The reader may verify that the following formulas closely approximate the data of Table 6.

Consumers Risk, β	Formula
.05	$n = \frac{29+16\,(Ac)}{10\,P_F}$
.10	$n = \frac{22+14\,(Ac)}{10\,P_F}$
.20	$n = \frac{16+12\,(Ac)}{10\,P_F}$

No attempt should be made to extrapolate the above formulas beyond the domain, or range of variables studied. Since the Reliability Engineer is generally not interested in high consumer risks nor higher probabilities of failure, further analysis, outside of the range shown, was not conducted. The tables, graphs and formulas provide a quick presentation of the relationships not apparent when looking at a binomial table.

	n at β = .2			n at β = .1			n at β = .05		
A_c \ $\doteq P_F$	.1	.2	.3	.1	.2	.3	.1	.2	.3
0	16	8	5	22	11	7	29	14	9
1	29	14	9	38	18	12	46	22	14
2	42	21	14	52	25	16	61	30	19
3	54	27	18	65	32	21	76	37	24
4	66	33	21	78	38	25	89	44	28
5	78	39	25	91	45	29	> 100	50	33

If A_c or fewer items fail out of n items tested, the probability of failure, P_F, is equal to or less than the amount shown, for the given consumers risk, β.

Table 6

Sample Size VS Acceptance Number

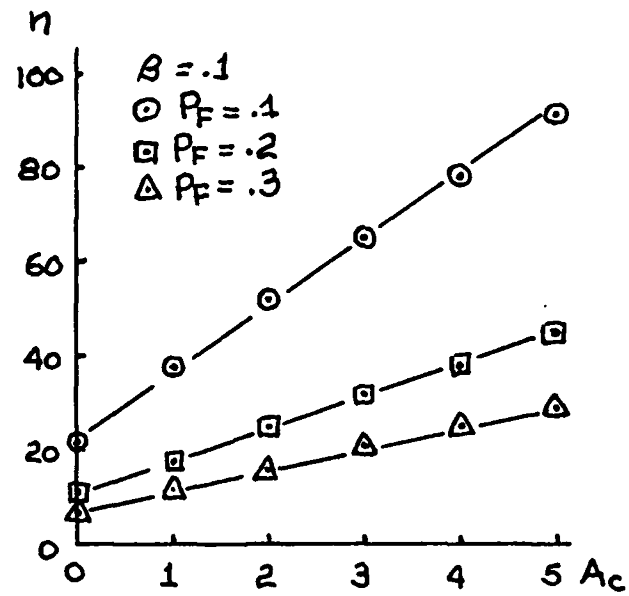


Figure 6

Sample Size VS Acceptance Number - Variable P_F

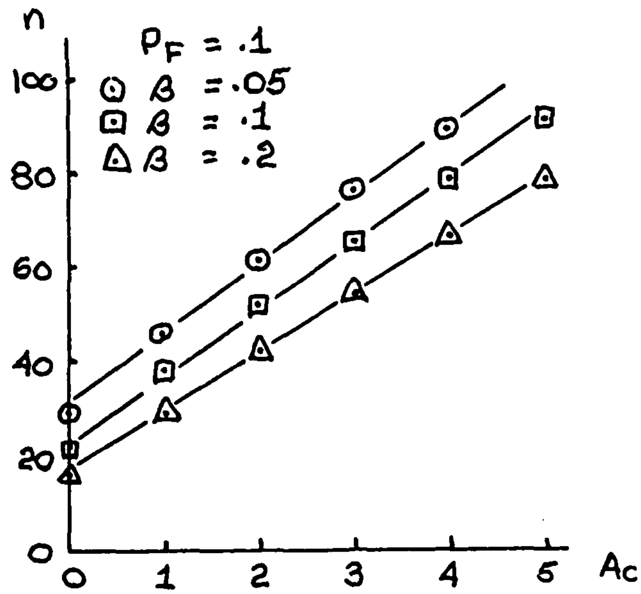


Figure 7

Sample Size VS Acceptance Number - Variable Consumer Risk

VIII Time Considerations in Reliability Testing

One shot devices make an important class of nonelectronic components but not the majority case. Far more often the test engineer is interested in affirming that the vast majority of components are operable for a given period of time. He thus specifies that the equipment must operate at a specified reliability for a specified period of time, usually corresponding to the expected operating time. He may also specify certain environmental conditons as well as sample size, rejection number, and test termination conditions.

It is possible to test a complete sample to failure, thus being able to determine, within predetermined accuracy, the mathematical model which best describes the manner in which the components fail over time. Using the model, the reliability can be determined at any desired time by estimating the proportion of all components which the model shows will survive to that time. The difficulty with this approach is the extreme amount of time that the average component requires for testing prior to failure. This is especially true since the engineer desires very high reliability in order of .95-.999 over the expected operating time. For most failure distributions, time to the final sample failure will be considerably higher than the operating time. Since time is money, test costs will be high. Also, time may not be available to wait for the final failure to make a decision. This manual approaches reliability demonstration testing from the viewpoint of minimizing test time.

Assuming no knowledge of the true failure distribution, a reliability analysis can be performed similar to that for one-shot devices by measuring the number of successes (components that do not fail) when each item in a sample is tested for a given time period. The attribute that we are then testing is the ability to operate successfully for the given time. As shown previously, sample size can be reduced at the expense of increasing the consumer's risk. Also, for a given consumer's risk, the sample size reduces

with an increase in the probability of failure, which increases with test time.

If the assumption of no knowledge regarding the failure distribution is relaxed to assume that the probability of failure increases directly with time, the sample size can be decreased inversely to increasing the time that each item in the sample is tested. This can be seen in the formulas of the preceding section by noting, that for any given Ac, the product of the sample size times the probability of failure is constant. Thus, if the probability of failure increases directly with time, the total test time, meaning the number of items tested multiplied by the time tested, will be constant. The cost of the test will thus be the same except for setup cost. Usually, because of high setup costs and limited test equipment, a longer test is conducted on a few items rather than a short test of many items.

Normally, the reliability engineer is interested in high reliability throughout the normal operating period. Since the expected probability of failure is low, testing only for the normal operating period requires a very large sample with prohibitive setup costs. The solution to this dilemma is to test a smaller sample for a longer period, at which time a much higher probability of failure is expected. If there is knowledge of how the failures occur with time, the engineer can project backwards to "prove" the reliability at the earlier normal operating period. Some knowledge about how the items fail over time can be gleaned from observing the times to failure of a portion of the sample even though all items are not tested to failure. If the sample is tested well beyond the normal operating time, the engineer may still be confident the test met his reliability goals.

A plot of the proportion of the sample which fails as the test progresses is known as a cumulative failure distribution. Figure 8 shows a plot of the statistic f/(n+1) versus time for a test of nine items along with a continuous curve interpolation of the data.

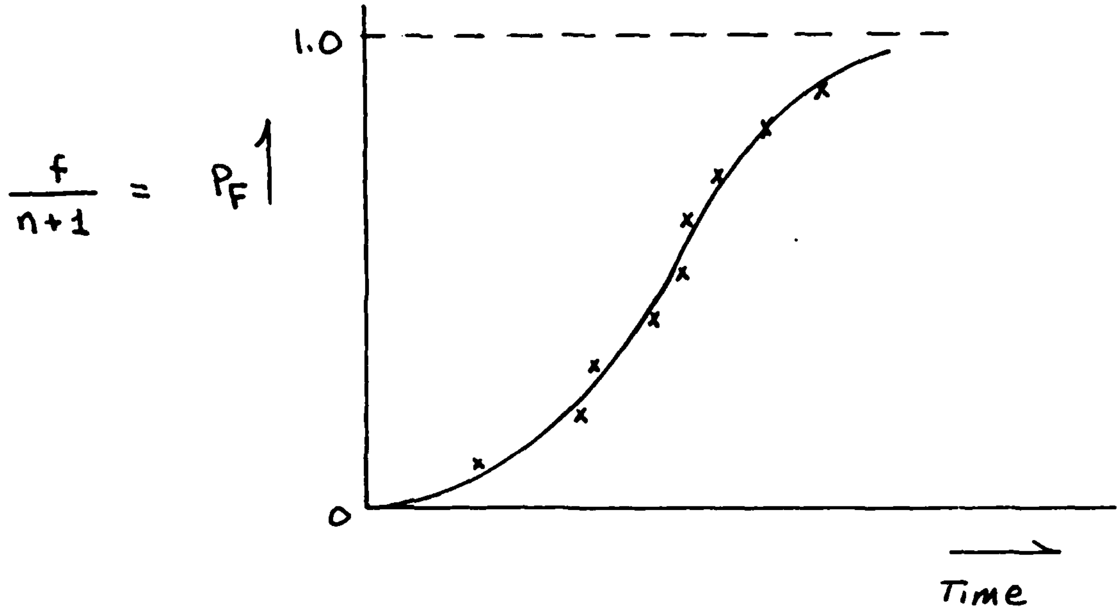


Figure 8
A Cumulative Failure Distribution

To correct for the small size, n+1 is used as the denominator where n is the number of items tested and f is the number of total failures at the time of the last failure, t. Frequently, the same data is analyzed by dividing the total time into increments and plotting the proportion of failures which occurred during each increment. This is known as histogram. Its smoothed or continuous version, when normalized so that the area under the curve is one, is called a failure probability density function or simply density function (Figure 9).

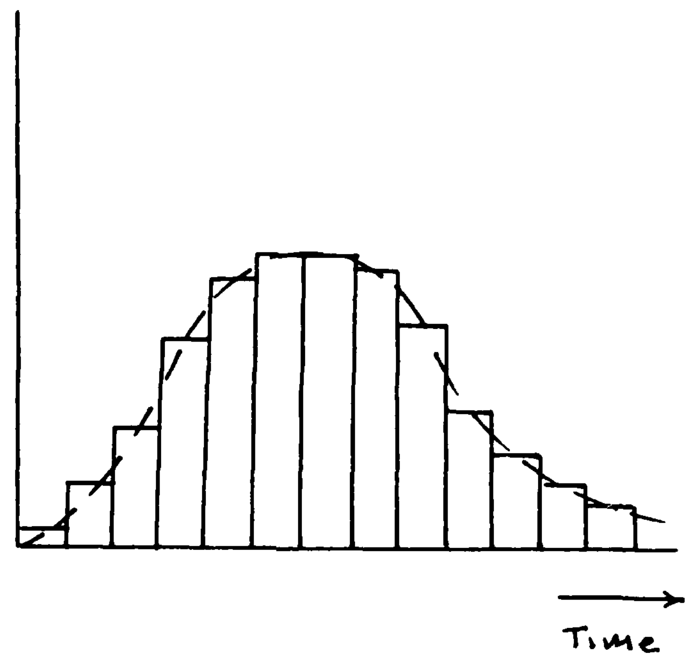


Figure 9
A Failure Density Function

By convention, the area under a failure density function is always established as unity since probability is defined as a number between zero and one. The integral of a failure density function is the cumulative failure distribution. Thus the height of the failure density function curve at any given time is the differential or slope of the cumulative failure distribution at that same time. Some writers use the terms "exponential distribution" or "Weibull distribution" to refer to the survival function which is one minus the cumulative failure distribution. Carefully review the writer's use of terms when using the reference material.

Rarely is reliability demonstrated by using the binomial test and

then terminating the test as early as the expected operating time. This is because the desired low probability of failure at this time requires too large of a sample. Rather, the testing consists of two efforts. One is an effort to define the nature of the failure distribution which represents the "true" or "parent population" failure distribution for the component. The other effort is a definition of the minimal test in terms of time and numbers tested, depending upon costs, to adequately demonstrate the desired reliability. The procedure is to test to some point, t_2, necessary to define a cumulative failure distribution precisely enough to be confident that the reliability at some earlier point, t_1, meets a desired objective. The cumulative failure distribution may be thought of as an envelope which describes the least acceptable of the many possible parent cumulative failure distributions from which the sample was taken. Demonstration consists in confirming from the test data that the process meets or exceeds this least acceptable failure distribution (Figure 10).

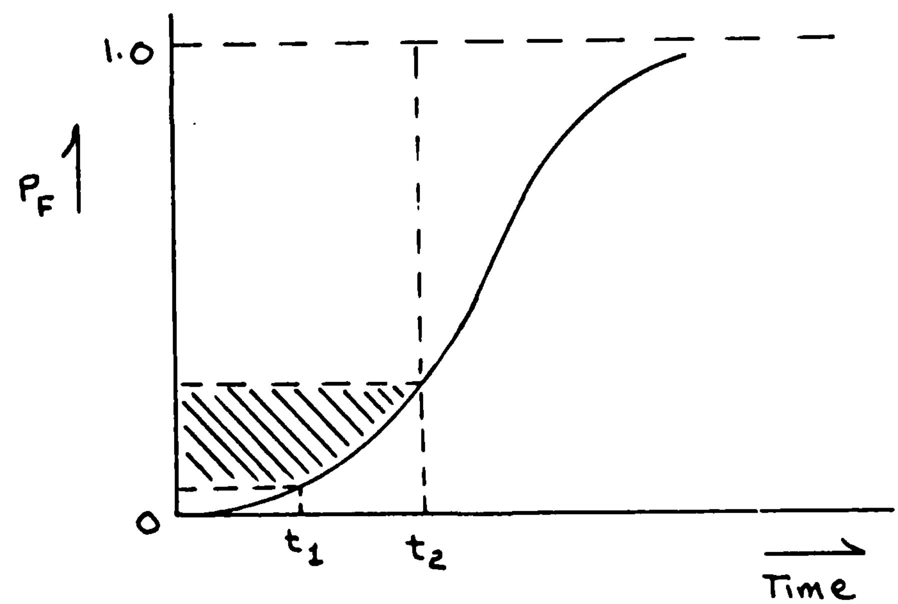


Figure 10
Defining the Cumulative Failure Distribution
from 0 to t_2 Defines P_F at t_1

IX How The Failure Distribution Affects Test Termination Time

For reasonably small sample sizes, it will be necessary to test until the acceptable probability of failure is in the order of $P_F = .20$. Since the desired reliability through an entire operating cycle is likely to be in excess of R = .95 (P_F=.05), each item in the sample must be tested for several times its anticipated operating time. This is especially true when testing items are expected to fail exponentially. It will be less true when the failure rate increases with time. Since the rate of failure is expected to increase with time for nonelectronic components, it may not be necessary to test a nonelectronic for as long (beyond its expected operating cycle) and still be confident of its reliable performance during its expected operating cycle. For instance, a normal distribution with a mean time to failure of 250 hours and with a variance of 40 percent of the mean (100 hours), will exhibit a reliability of R=.95 at 85.5 hours. Let's assume that a reliability of .95 at 85.5 hours is our requirement. Testing for an expected test termination time at which the probability of failure is 20 percent (P_F=.2) requires testing for 165.8 hours. An exponential distribution with a mean time to failure of 1666.9 hours also exhibits a reliability of R =.95 at 85.5 hours. However, testing for an expected test termination time at which P_F=0.2 requires testing for 372 hours. This is an increase in test time of 124 percent.

In the above example, comparison of a normal distribution with an exponential was made because of the familiarity of most readers with either of these distributions. Actually, use of the normal distribution is not appropriate for the type of problem in question. Two problems arise. First, the normal distribution is an infinite distribution extending from both sides of the mean. The distribution thus predicts some finite probability of failure prior to operation, which should be physically impossible. Also, the normal distribution is defined by two parameters, the mean and the variance. In time or failure terminated (truncated) tests,

there is not enough data to estimate either parameter. This precludes using normal distribution as a failure model in truncated tests.

A particular form of the Weibull distribution, which is also defined by two parameters, is quite flexible and can approximate a normal distribution in certain forms. With certain assumptions, a direct derivation of test termination times for changes in a single parameter is possible. For these reasons, the Weibull distribution will be the distribution of choice for further analysis.

The general form of the Weibull density function is:

$$f(x)=a \cdot b \cdot X^{(b-1)} \cdot e^{-a \cdot x^{b}}$$

Appendix 2 shows the derivation of a two parameter expression of the Weibull which is used herein to describe the Reliability Survival function.

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^{m}}$$

where: $R(t)$ = Reliability at time, t.

m = Shape parameter

θ = Scale parameter

If time is measured in terms of t/θ units, the Weibull becomes a single parameter function. In the special case, where $m=1.0$, the Weibull becomes the exponential distribution with θ equal to the mean time to failure. For other values of m, θ will not equal the mean time to failure. However, each will have the same reliability at $t=\theta$.

NOTE: To avoid confusion "mean time to failure" (MTTF) as defined in H-108 and "mean time between failures" (MTBF) as defined in MIL-STD-718C are equivalent as used in the exponential Reliability Survival function and as used in this paper. Consult MIL-STD-721C for a precise definition of terms.

Frequently, there is no knowledge regarding the nature of any component's failure distribution until some items have failed under test or usage. The exception to this is when the component is similar to another component for which a failure distribution is known. Procuring agencies typically require a test from MIL-STD-781C or H-108 for nonelectronic as well as electronic components. Later, the conservatism of this decision will be analyzed. For now, let us discuss the case where some failures have occurred in a time truncated test. First, let us consider reducing the test termination time as the shape parameter changes.

Figure 11 shows the reduction in test termination time, compared to an exponential distribution, as the shape parameter increases for a series of Weibull distributions. Each distribution meets a specified reliability requirement of $R_t = .95$ at time, t. Test termination time occurs at time, T, when $R_T = .80$ for each distribution. The general formula for test termination time as a proportion of the exponential test termination time is:

$$Q = \frac{T_m}{T} = \left[\frac{\ln R_t}{\ln R_T}\right]^{(1-1/m)}$$

where

R_t = required liability at time, t.

R_T = reliability at test termination time, T.

T = test termination time for an exponential distribution (m=1)

T_m = test termination time for a Weibull distribution with shape parameter, m.

Appendix 2 shows the derivation of the above formula.

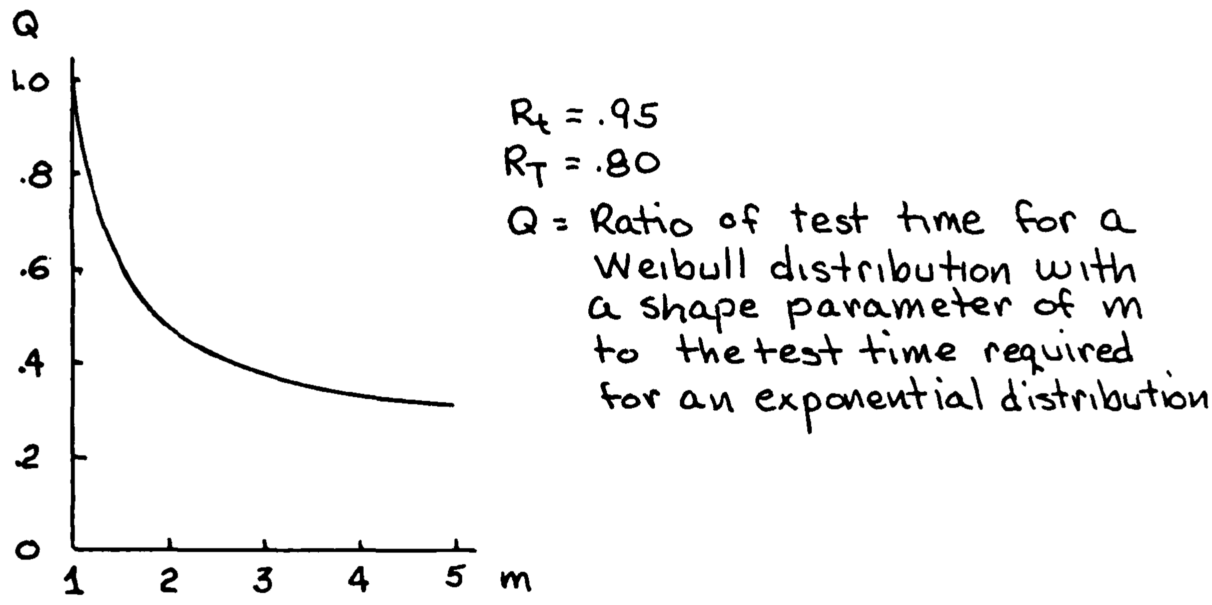


Figure 11

Test Time VS Weibull Shape Parameter

X HOW CONSERVATIVE IS THE "EXPONENTIAL ASSUMPTION"?

Although there is widespread agreement that non-electronic components are unlikely to fail in accordance with the exponential model, the ease with which the exponential test specifications can be applied perpetuates their widespread usage. In addition, the exponential model lends itself to predicting system reliability for a group of components. Without failures, there can be no knowledge as to the nature of the failure distribution. Still, there is sufficient data to predict the minimum reliability at the time of test termination, with or without assuming a certain failure distribution, assuming an exponential failure distribution entails certain risks which are the subject of this section. Table 7 shows the probabilities of failure for various distributions with equal median times to failure (time at which $P_F = .5$).

t/M	Rectangular	Normal $\mu=1.0, \sigma=.3$	Normal $\mu=1.0, \sigma=.4$	Exponential $m=1.0$	Weibull $m=1.5$	Weibull $m=2.5$	Weibull $m=3.5$
0	0	.00043	.00621	0	0	0	0
.01	.005	.00048	.00667	.00691	.00069	.00001	.00000
.02	.010	.00054	.00714	.01377	.00196	.00004	.00000
.03	.015	.00062	.00765	.02580	.00360	.00011	.00000
.04	.020	.00069	.00820	.02735	.00530	.00022	.00001
.06	.030	.00087	.00939	.04074	.01014	.00061	.00004
.08	.040	.00107	.01072	.05394	.01556	.00125	.00011
.10	.050	.00135	.01222	.06697	.02168	.00219	.00023
.15	.075	.00233	.01680	.09875	.03947	.00602	.00095
.20	.100	.00379	.02275	.12945	.06011	.01232	.00259
.25	.125	.00621	.03035	.15910	.08300	.02143	.00565
.30	.150	.00990	.04006	.18775	.10765	.03359	.01067
.35	.175	.01500	.05208	.21542	.13370	.04899	.01824

TABLE 7
Comparison of Failure Distributions

In the preceding table M = time at which P_F = .50. The Table lists the probability of failure for various distributions for times in proportion to the median time to failure. Notice that all probabilities of failure, for distributions other than exponential, are lower for each time computed than is the exponential. This is not universally true for all distributions. It is true for the distributions shown and will be generally true for any distribution where the hazard rate (probability of failure in the next small increment of time) is increasing with time. Figure 12 shows, for the Weibull distribution, how the hazard rate varies with time for various shape parameters.

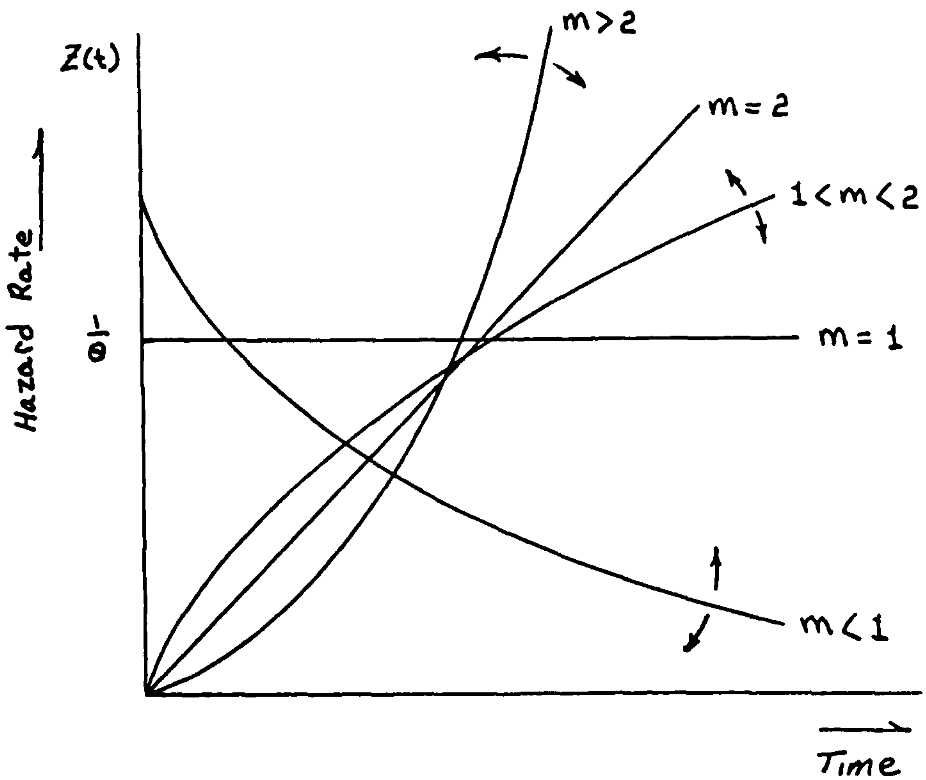


Figure 12
Weibull Hazard Rate VS Time

Since reliability is simply unity minus the probability of failure, the general rule is that, for any given reliability requirement (reliability specified at a certain time), distributions whose hazard rates increase with time will be more reliable than the exponential at less than the time for which the reliability requirement is specified and less reliable at more than that time. Of course, the true distribution may have a hazard rate which decreases with time, in which case the reverse would be true. Hazard rates decreasing with time are not expected for non-electronic components.

Figure 13, below illustrates this principle. Both the Weibull and the Exponential cumulative failure distributions have the same reliability at time, t. At times less than t, the Weibull distribution has a lower P_F (higher reliability since $R = 1-P_F$). At times greater than t, the reverse is true and the Weibull components will be less reliable than components failing exponentially which also have a reliability of R at time, t.

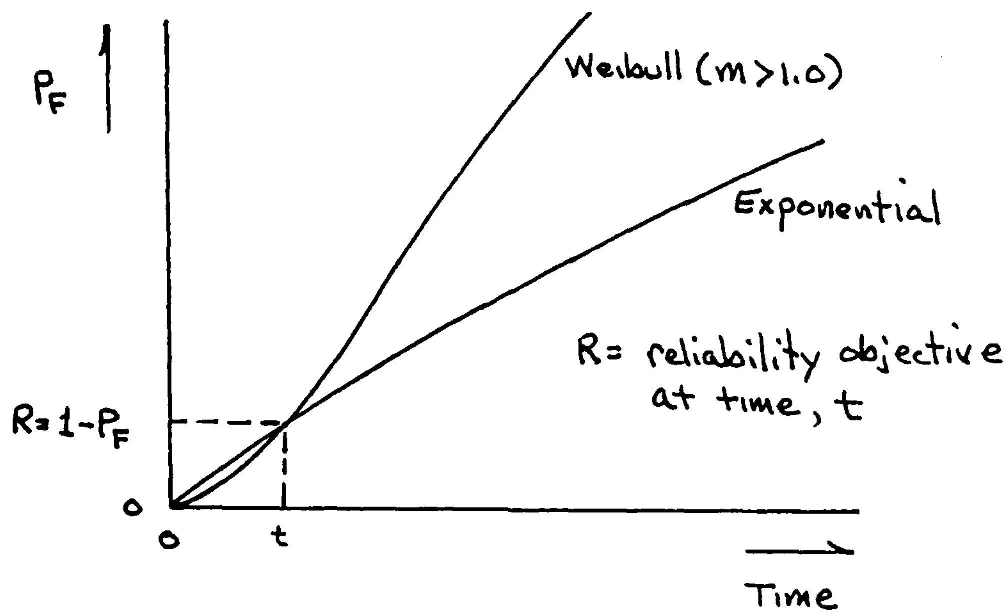


Figure 13
Failure Distributions VS Reliability Objectives

PART CLASS: BATTERY

TYPE: Nickel Cadmium

ENV	SPEC NUMBER PART NUMBER MANUFACTURER	CHARACTERISTICS	FAILURE RATE/10^6 HOURS				NUMBER FAILED	OPERATING HOURS ($x\ 10^6$)
			$\hat{\lambda}$	60% UPPER SINGLE-SIDED CONFIDENCE	60% CONFIDENCE INTERVAL LOWER	60% CONFIDENCE INTERVAL UPPER		
SAT		1 Cell	---	0.0216	---	---	0	42.398
SAT		1 Cell	0.092	---	0.053	0.154	4	43.465
GRF		2 Cell	3.290	---	2.294	4.664	8	2.431
GRF		3 Cell	---	---	---	---	0	0.143
GRF		4 Cell	0.596	---	0.454	0.783	13	21.806
GRF		5 Cell	2.219	---	2.059	2.437	136	61.277
GRF		6 Cell	2.471	---	1.265	4.546	3	1.214
GRF		8 Cell	---	0.0669	---	---	0	13.686
GRF		10 Cell	1.014	---	0.707	1.438	8	7.886
GRF		20 Cell	0.709	---	0.363	1.304	3	4.233
GRF		21 Cell	---	---	---	---	0	0.114

Figure 14

Failure Rate Data

If failures have occurred, it is frequently possible (see Chapter XI) that information can be gleaned regarding the nature of the distribution. The case of interest, then, is when one or no failures have occurred. The engineer may accept the test item and then go on to estimate a MTBF, assuming an exponential failure distribution. This is generally done by estimating the lower confidence limit to some explicitly stated consumer's risk. Figure 14 is excerpted from reference 14 and shows how the data may be presented. In the case of figure 14 the lower one sided confidence level for a MTBF estimate corresponds to the upper one sided confidence level for the failure rate (which is the reciprocal of the mean time to failure).

As an example, consider a test in which 50 items are tested for 100 hours each and no items fail. Without knowledge of the type of distribution, interpolation of the binomial tables (as excerpted in Table 8) for a cumulative probability of 0.4 (consumers risk) provides a one sided sixty percent confidence limit for the probability of failure at 100 hours of:

$$P_F = .01 + (.02 - .01)\cdot\left[\frac{.605006 - .400000}{.605006 - .364170}\right] = .0185$$

Subtracting from one gives a reliability at 100 hours of:

$$R = 1 - .0185 = 0.9815$$

The associated mean time to failure, assuming an exponential failure distribution is:

$$R = e - \frac{100}{\Theta}$$

$$\Theta = \frac{100}{-\ln(.9815)} = 5355 \text{ hrs.}$$

Use of the χ^2 table in Appendix 1 provides a similar estimate of Θ as:

$$\Theta = \frac{2T}{\chi^2_{\nu,p}} = \frac{2T}{{}^2{}_{2,.6}}$$

$$= \frac{2(50)(100)}{1.83} = 5464 \text{ hrs.}$$

Where: r = number of failures = 0

T = total test time = 50 (100)

p = confidence level/100 = .6

ν = degrees of freedom = 2r+2

N = 50

p = .01

x	Individual Term	Cumulative (x or less)
0	.605006	.605006
1	.305559	.910565
2	.075618	.986183
.		
.		
.		

p = .02

x	Individual Term	Cumulative (x or less)
0	.364170	.364170
1	.371602	.735771
2	.185801	.921572
.		
.		
.		

Table 8

Cumulative Binomial Probabilities

Source: Reference 7

It is important to remember that, unless failures in fact occur exponentially, knowlege as to the reliability exists only at 100 hours. Independent of analysis into the nature of the failure distribution, there can be no knowledge as to how the component may fail with additional testing. Table 9 shows the reliability for Weibull distributions with various shape parameters as time varies from the 100 hours used in the above example.

Time	Reliability					
T	Exponential 1.0	Weibull 1.5	2.0	3.0	4.0	Shape Parameter
70	.9870	.9891	.9910	.9936	.9955	
80	.9852	.9867	.9881	.9905	.9924	
90	.9833	.9842	.9850	.9865	.9878	
100	.9815	.9815	.9815	.9815	.9815	
110	.9797	.9787	.9777	.9755	.9730	
120	.9778	.9758	.9735	.9682	.9620	
130	.9760	.9727	.9689	.9598	.9481	

Table 9

Reliability VS Shape Parameter

While the results of Table 9 are expressed in reliability terms, it is important to recognize that, at 130 hours, the probability of failure has increased by 14% for m = 1.5 to 116% for m = 4.0. Without knowing the true failure distribution, the engineer certainly doesn't want to be predicting the reliability at times exceeding 100 hours. But, what about the opposite side of the coin? At 70 hours, the probability of failure has decreased by 16% for m = 1.5 to 65% for m = 4.0 as compared to that probability of failure to be expected if failures occurred exponentially. Here, the engineer may reasonably expect that an estimate of the reliability at less than 100 hours using the exponential model,

$$R(t) = e^{-\frac{t}{\theta}}$$

is conservative. Conservative in the sense that, having passed the above test, the reliability will at least equal the computed figure. This will be the case unless the true distribution is skewed even more to the left than is the exponential. A Weibull distribution with a shape parameter of m = 0.5 is an example of such a skewed distribution. Since mechanical component failures tend to increase with time, due to wear, this type of distribution is not expected.

As previously discussed, for reasonable sample sizes, test termination time will exceed the normal operating time. Figure 15 graphs a comparison by percentage of the probabilities of failure of two Weibull distributions (m = 2.0 and m = 0.5) both of which have passed a test demonstrating a reliability of 0.8 (P_F = 0.2) at time, T. The time extends from .1T to 1.3T. The risk of having a true distribution with a declining hazard rate is apparent.

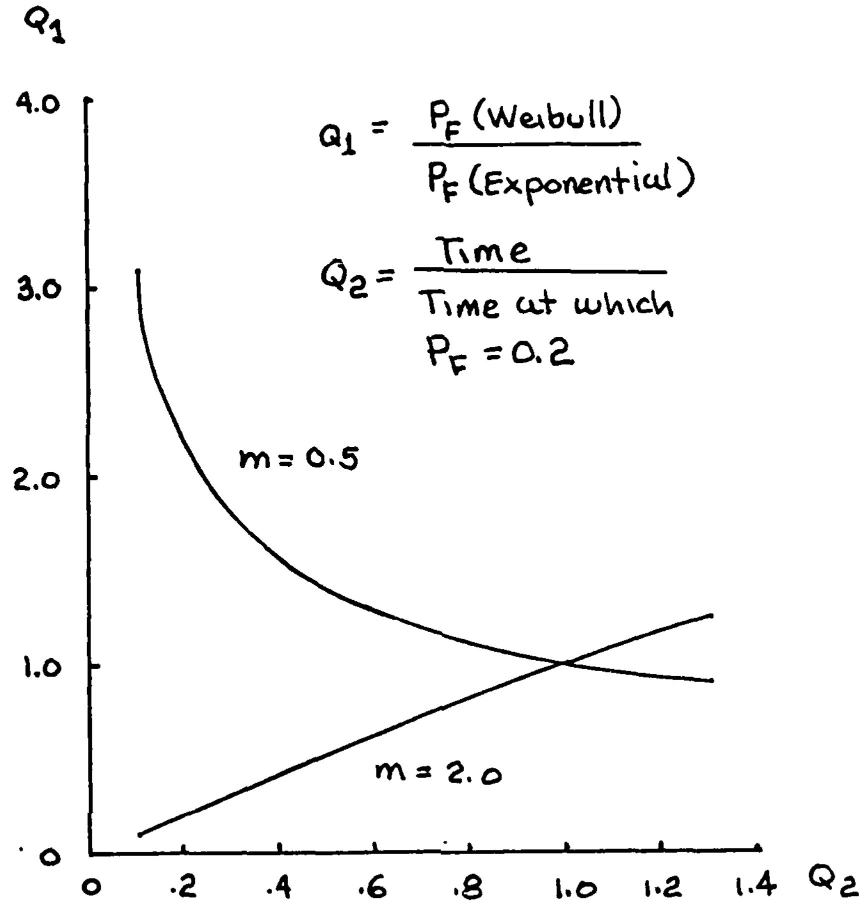


Figure 15
Failure Probability VS Shape Parameter

Although a distribution in which the hazard rate decreases with time is not expected, it is best to rule out the possibility. This is not possible unless there are sufficient failures to discriminate at least between a rising or decreasing hazard rate. Lacking data, the engineer must use his engineering judgment regarding the nature of the expected failure modes.

If failures have occurred, a simple discrimination between an increase or a decrease can be performed by using the chi-square table of Appendix 1 and a 2X2 contingency table. This is done by dividing the failures into two groups depending upon whether the failure occurs earlier or later than that time at which one half of the failures may have been expected to have occurred and testing for a preponderance of early or late failures. The 2X2 contingency table is shown below (Figure 16).

a	b	a+b
c	d	c+d
a+c	b+d	n

Figure 16
A 2X2 Contingency Table

A short-cut method for computing chi-square is as follows:

$$\chi^2 = n(ad-bc)^2/K$$

where $n = a+b+c+d$

and $K = (a+b)(c+d)(a+c)(b+d)$

with one degree of freedom ($\nu = 1$)

This formula is an approximation but is adequate for our purposes.

For use in failure time comparisons, the 2X2 contingency table is of the form:

H	L	H+L
$\frac{H+L}{2}$	$\frac{H+L}{2}$	H+L
$\frac{3H+L}{2}$	$\frac{H+3L}{2}$	2(H+L)

Where: H = number of failures occurring beyond T/2

L = number of failures occurring in less than T/2

T = test termination time

As an example, assume that three out of five failures occur at greater than one half of the time to termination.

$$\chi^2 = \frac{10(3\cdot 2.5 - 2.5\cdot 2)^2}{(5\cdot 5\cdot 5\cdot 5.5\cdot 4.5)}$$

$$= .1010$$

3	2	5
2.5	2.5	5
5.5	4.5	10

Reference to appendix 1 shows a relatively low (slightly greater than twenty five percent) confidence that the probability of failure in the latter half of the test termination time exceeds the probability of an earlier failure. This data does not provide reason for rejecting the exponential model.

Suppose, however, that four out of five failures had occurred prior to one half of the test termination time. In this case:

$$\chi^2 = \frac{10(1 \cdot 2.5 - 4 \cdot 2.5)^2}{(5 \cdot 5 \cdot 6.5 \cdot 3.5)}$$

$$= .9890$$

1	4	5
2.5	2.5	5
3.5	6.5	10

Reference to appendix 1 shows a quite high (approximately seventy percent) confidence that the probability of failing early, in less than half of the test termination time, exceeds failing in the latter half. Results like this would certainly require more testing and analysis prior to deciding to use an exponential model.

XI DATA ANALYSIS TO REVISE TEST ACCEPTANCE CRITERIA

As discussed previously, the key decision facing the test engineer, once he has selected a sample size, is how long to test. Unfortunately, other than assuming an exponential failure distribution, there is no basis to select a test termination time. However, with each failure, assuming that failures do indeed occur, information can be gleaned to define how the failures are occuring over time. For instance, Appendix 2 discusses a special Weibull probability paper used to plot failure percentages versus time in order to define a Weibull failure distribution.

With computers and sophisticated curve fitting programs, it is possible to determine the most likely estimate of the reliability function. Lacking such sophisticated analysis tools, this paper suggests analyzing the first and last failure based upon the assumption of a Weibull reliability function. As developed in Appendix 2, the formula for calculating the sample shape parameter is:

$$m = \frac{\ln\ln\left[\frac{1}{R(t_2)}\right] - \ln\ln\left[\frac{1}{R(t_1)}\right]}{\ln(t_2) - \ln(t_1)}$$

where:

t_2 = last failure time

t_1 = first failure time

$$R_{(t)} = 1 - \frac{r}{n+1}$$

r = number of failures at time, t

n = sample size

This formula can be quickly solved with any calculator capable of determining natural logarithms. First plot the failure points to assure that a straight line from the first to the last point is reasonably representative of all the failures.

Without prior knowledge as to the distribution, begin with the assumption that failures will occur exponentially. Provided that failures do occur, application of the above formula may still provide confidence that the component is sufficiently reliable even though it has failed the exponential test. For instance, Figure 11 shows, for a reliability requirement of R = .95, testing of components from a Weibull population with a shape parameter of m = 2.0 may be terminated in only half the time required for testing an exponential distribution. Thus, if thirty components were under test and six items failed, leading to a decision to stop the test and to reject the lot, further analysis of the six failures may lead the engineer to lower his test termination time and to accept the lot based upon a lower number of failures that may have occurred by the earlier time.

In order to gain some confidence in using the above formula, simulation studies were run in which random numbers were generated between .00001 and .99999. These were then transformed into failure times according to the Weibull reliability model for a variety of shape parameters. One thousand simulated samples were then generated for a given sample size and the sample shape parameter was calculated from the first and last failure in accordance with the above formula for a variety of rejection numbers. The proportion of the estimated shape parameters falling below a given value was then deter-

mined in relation to the number of simulations (1000).

Results are presented in Appendix 4. As an example, refer to the following Table 10, taken from Appendix 4, and Table 11, which summarizes selected data points for a variety of sample sizes.

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 30
POPULATION SHAPE PARAMETER 1

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.629	.715	.758	.791	.814	.828
3	.739	.833	.888	.911	.932	.949
4	.78	.881	.923	.948	.972	.978
5	.817	.915	.949	.975	.985	.991
6	.85	.933	.966	.985	.991	.995
7	.855	.948	.978	.99	.995	.998
8	.872	.96	.984	.996	.998	.999
9	.889	.967	.988	.995	.998	.999
10	.902	.976	.991	.998	.999	1

Table 10

Proportion of shape parameter estimates below specified levels

r = 4

Population Shape Parameter = 1

m \ n	10	20	30	40	50
1.5	.788	.759	.780	.773	.776
2.0	.886	.856	.881	.872	.867
2.5	.941	.922	.923	.918	.927
3.0	.962	.946	.948	.941	.953
3.5	.973	.965	.972	.959	.972
4.0	.980	.977	.978	.966	.976

Table 11
Proportion of shape parameter estimates below specified levels

Each column in Figures 10 and 11 presents the proportion of the sample shape parameters from the 1000 simulations which fall below a cut-off value, labeled as M in table 10 and m in table 11. Since the simulation failure times are from a known population shape parameter (M = 1.0 in this case), this proportion is a measure of the probability that, upon randomly selecting a small sample with failures and computing a sample shape parameter, the result will be less than the cut-off value. Thus having selected thirty items at random and computed a sample shape parameter of 2.0 on the fifth failure, one is approximately 91.5% confident that the population shape parameter is at least 1.0. Table 11, also simulated from a Weibull distribution with a shape parameter of one, shows that confidence does not increase significantly as the sample size increases for a given rejection number.

Appendix 4 presents a series of tables for simulated truncated tests, similar to Table 10, for parent Weibull populations with shape parameters that vary from 1.0 to 1.5. Tables like those presented in Appendix 4 are necessary to provide some estimate of how probable it is that data analyzed from a small sample may lead to an erroneous conclusion about the parent population.

As an example of using Appendix 4, consider the engineer who chooses a time terminated test from Table 2C-3 in H-108 (See Table 12). The supplier is willing to accept a ten percent risk (α = 0.1) that his bearings will fail the test should the bearing have a MTBF as low as 3000 hours. The engineer will accept a ten percent risk (β = 0.1) that bearings with a MTBF as low as 1000 hours will pass the test. He will accept no higher risk because he wants a reliability of at least 0.95 at 50 hours. Lacking information, and as a matter of convenience, the engineer chooses H-108. He will test 34 bearings, chosen at random, for 300 hours. If six bearings fail, the lot will be rejected.

Table 2C-3—Continued

θ_1/θ_0	r	T/θ_0 1/3 n	1/5 n	1/10 n	1/20 n	r	T/θ_0 1/3 n	1/5 n	1/10 n	1/20 n
		$\alpha=0.10$		$\beta=0.01$			$\alpha=0.25$		$\beta=0.01$	
2/3	77	238	369	699	1358	52	168	261	496	965
1/2	26	73	112	210	407	17	51	79	149	289
1/3	11	27	40	75	145	7	19	29	54	105
1/5	5	10	14	26	51	3	6	10	18	36
1/10	3	5	7	12	23	2	3	5	10	20
		$\alpha=0.10$		$\beta=0.05$			$\alpha=0.25$		$\beta=0.05$	
2/3	52	156	242	456	886	32	101	156	296	576
1/2	18	48	73	137	265	11	31	48	91	177
1/3	8	18	27	50	97	5	12	19	36	69
1/5	4	7	10	19	36	2	3	5	10	20
1/10	2	2	3	6	11	2	3	5	10	20
		$\alpha=0.10$		$\beta=0.10$			$\alpha=0.25$		$\beta=0.10$	
2/3	41	121	186	351	681	23	71	110	207	403
1/2	15	39	59	110	213	8	22	33	63	123
1/3	6	12	18	34	66	4	9	14	27	52
1/5	3	5	7	12	23	2	3	5	10	20
1/10	2	2	3	6	11	1	1	1	3	6
		$\alpha=0.10$		$\beta=0.25$			$\alpha=0.25$		$\beta=0.25$	
2/3	25	69	107	201	389	12	34	53	101	196
1/2	9	21	31	58	113	5	12	19	36	69
1/3	4	7	10	19	36	2	3	5	10	20
1/5	3	5	7	12	23	1	1	1	3	6
1/10	2	2	3	6	11	1	1	1	3	6

No operating characteristic curves are provided for these sampling plans. However, two points on the OC curves $(1, 1-\alpha)$ and $(\theta_1/\theta_0, \beta)$ are given.

Table 12

H-108 Test Plan (Reference 10)

The test commences. The first bearing from the sample fails at 147 hours. The sixth bearing fails at 250 hours. The test is a failure and a lot of precision bearings worth $500,000 is about to be rejected. The engineer decides to look more closely at the data, especially since the bearings are not expected to be operated over fifty hours.

Calculating an estimated Weibull shape parameter from the data gives:

$$\text{Est. } M = \frac{\ln\ln\left(\frac{1}{1-6/35}\right) - \ln\ln\left(\frac{1}{1-1/35}\right)}{\ln(250) - \ln(147)}$$

$$= 3.52$$

Perusing Appendix 4 shows that, for n=30, r=5 (chosen because 5/30 closely approximates 6/34), and an estimated shape parameters of 3.5, there is a very high probability (.941) that the true population Weibull shape parameter is not lower than 1.5. In fact, the probability that a sample of 30 from a population with a shape parameter of 1.5 would provide a test statistic of 3.5 on the fifth failure is approximately 1-.941=.059. This is only an approximate probability which could be more accurately defined by increasing the number of simulations above 1000.

As can be seen from Table 10, there is also a probability (1-.985=.015) that the results may be obtained from an exponential distribution (Weibull shape parameter equal to one). Choosing the higher shape parameter of 1.5 is a matter of judgment. Plotting the failures on Weibull graph paper can also add confidence to the judgment. If the final or first failures do not appear representative, the engineer may choose to reject a data point and recompute the estimated shape parameter. This decision would result in reducing 'r' (the final failure number) by one for each data point rejected.

The true Weibull shape parameter for the population from which the sample was taken has thus been judged to be at least 1.5. In selecting the test plan from H-108, the engineer had chosen to terminate the test when the reliability equalled .7408, as computed below:

$$R_t = e^{-\frac{300}{1000}}$$

$$= .7408$$

For a shape parameter of 1.5 and the same reliability requirement (R = .95 at 50 hours) an equivalent test would be terminated at 166 hours, prior to the sixth failure, as shown below:

$$\frac{T_m}{T} = \left[\frac{\ln R_t}{\ln R_T}\right]^{1-\frac{1}{m}}$$

$$T_m = T\left[\frac{\ln R_t}{\ln R_T}\right]^{1-\frac{1}{m}}$$

where:

R_t = Desired reliability at time, t.

R_T = Reliability of an exponential distribution at test termination time, T.

T_M = Test termination time for Weibull distribution with parameter, M.

then:

$$T_{1.5} = 300 \left[\frac{\ln .95}{\ln .7408}\right]^{(1-1/1.5)}$$

$$= 300\ (.1708)^{1/3}$$

$$= 166 \text{ hrs}$$

There is little question that the bearing meets the minimum reliability requirement. However, the exact confidence level is now known. This is because we are not certain of the true population failure distribution. In addition, the exact simulation (6/32 as opposed to 5/30) was not performed; nor is a simulation of one thousand really accurate to three digits. Nevertheless, the engineer's confidence is much better than simply multiplying the sample size confidence level times the confidence in an estimate of the population shape parameter (.90 X .941 = .85). This is because of the relatively long time to first failure, as will be shown in the following section.

What about the desired reliability implied by a MTBF of 3000 hours? At 50 hours, the desired reliability can be computed as:

$$R = e^{-\frac{50}{3000}}$$

$$= .9835$$

Terminating the test at 300 hours meant that the engineer was testing for a period in which the probability of failure would be no higher than:

$$R_t = e^{-\frac{300}{3000}}$$

$$= .9048$$

For a shape parameter of 1.5 and the same reliability requirement (R = .9835 at 50 hours) an equivalent test would be terminated at 166 hours, the same conclusion as with the lower MTBF.

$$\text{e.g.} \quad T_{1.5} = 300 \left[\frac{\ln .9835}{\ln .9048} \right]^{(1 - 1/1.5)}$$

$$= 300(.17098)^{1/3}$$

$$= 166 \text{ hrs}$$

The real question is one of mission. How long will these bearings be expected to operate? Fifty hours is one thing, five hundred quite another. The minimum reliability requirement should be established at the upper time limit for which the bearing is expected to operate.

The above principle is especially critical when considering a reduced test termination time based upon analysis of limited data. The test termination time is directly related to the time at which the reliability objective is specified. Beyond this specified time, even if the exactly correct Weibull shape parameter is chosen, the true reliability will be less than that expected for the exponential failure model. This is inherent within the nature of an increasing hazard rate. It is, therefore, extremely important that the specified reliability objective be at the end of the expected operating time. If not, techniques other than truncated tests must be used.

XII TIME TO FIRST FAILURE

The probability of a perfectly balanced coin being flipped three times in a row, with all three flips coming up heads, is $(1/2)^3 = 1/8$ or .125. If this happened, the statistician would say that he is 87.5 percent confident (1 - .125) that the probability of obtaining a head on any one flip is at least 0.5. He accepts a consumer risk of .125 that his conclusion is wrong. This conclusion is based upon the evidence from only three flips. What if he continues his test to ten flips and each flip also turns up heads? The probability of obtaining a head on any one flip is 0.5 and the probability of ten flips in a row coming up heads is .5 times itself ten times or $(.5)^{10} = .001$. Given this result, he is 99.9 percent confident that the probability of obtaining a head is at least 0.5. This conclusion is very conservative with a consumer risk of only .001. At this point, he may suspect that the coin has a better than even chance of coming up heads. How low could the probability of obtaining a head on any one flip be, if he is willing to accept the same consumer risk of 0.125, given the result of ten flips in a row coming up heads? The answer to this is the reverse of the prior process and requires taking the tenth root of the acceptable consumer risk, or $(.125)^{1/10}$. The tenth root of .125 equals .8123. He can then say that he is 87.5 percent confident that the probability of a head is at least .8123. It may be higher. But, if it is truly less, his test result of ten flips in a row with no tails is a less common occurrence and, given the evidence, to be expected less than 12.5 percent of the time. A 12.5 percent chance of being wrong (consumer risk) is accepted by the engineer in concluding that the probability of a head on any one flip is no less than .8123.

The reasoning in the above paragraph would be the same if the question were one of squib reliability. If ten squibs, selected at random, are fired with no failures, the engineer is 87.5 percent confident that the reliability of the squib is at least .8123. The same conclusion prevails

with ten gear boxes, selected at random, tested for 1,000 hours with no failures. The engineer is 87.5 percent confident that the reliability of the gearbox is at least .8123 when operated for no more than 1,000 hours. In each test, there must be no failures and the conclusion is based upon the nth (n = number tested) root of the desired consumer risk, in this case the tenth root of 0.125. In the case of a more traditional consumer risk, say 0.10, the engineer may state that he is 90 percent confident that the reliability is at least .7943, the tenth root of the consumers risk, 0.10, being .7943.

In the preceding section, we discussed a test where 34 items were tested with the first failure occurring at 147 hours. Just prior to 147 hours a situation existed analogous to the coin example. Thirty-four items had been tested with no failures. Accepting a consumer risk of 0.10, and asking how low the reliability may be, provides an answer of 0.9345, the thirty-fourth root of 0.10. The engineer can say that, shortly prior to 147 hours, he is ninety percent confident that the bearing reliability is at least 0.9345. This conclusion is independent of how the bearings fail with time and is thus called a non-parametric estimate of the reliability.

The engineer originally wanted a reliability of no less than 0.95 at 59 hours. Data analysis indicated that the sample may be failing in accordance with a Weibull failure distribution where the shape parameter, m, equals or exceeds 1.5. Using the Weibull reliability model:

$$R_{147} = .9345 = e^{-\left(\frac{147}{\theta}\right)^{1.5}}$$

$$\ln(.9345) = -\left(\frac{147}{\theta}\right)^{1.5}$$

$$\left[-\ln(.9345)\right]^{\frac{1}{1.5}} = 147/\theta$$

$$\theta = \frac{147}{\left[-\ln(.9345)\right]^{\frac{1}{1.5}}} = 885$$

At 50 hours, the reliability would then be:

$$R_{50} = e^{-(50/885)^{1.5}} = .9867$$

well above the minimum reliability requirement of .95.

What if the sample is from a population failing exponentially? Then

$$R_{147} = .9345 = e^{-147/\theta}$$

$$\theta = \frac{147}{-\ln(.9345)} = 2170$$

at 50 hours the reliability would then be:

$$R_{50} = e^{-50/2170} = .9722,$$

also well above the minimum reliability of .95.

Another way of looking at the time prior to the first failure is to ask how long a test must run with no failures in order to provide confidence that a given Weibull distribution has met a specified reliability requirement. The general formula, as developed in Appendix 2, for answering this question is:

$$\frac{T}{t} = \left[\frac{\ln(\beta)}{n \cdot \ln(R)}\right]^{\frac{1}{m}}$$

where:

T = Time for which no failures are to occur.

t = Time for which the reliability requirement is to be met.

β = Consumer risk

n = Number of randomly selected sample items that are to be tested.

R = Specified reliability requirement at time, t.

m = Weibull shape parameter

Testing 34 items from a population that is failing exponentially would thus provide a criteria for test termination with no failures at 67 hours, given a reliability requirement of 0.95 at 50 hours. This is shown below.

$$T_{50} = 50\left(\frac{\ln .1}{34 \ln .95}\right)^{1.0} = 67 \text{ hours}$$

It would thus have been possible to terminate the test and accept the proposition that the equipment's hazard rate was either constant (exponential) or increasing with time (Weibull with $m > 1.0$).

Appendix 6 presents Tables of T/t (identified as R) computed for a variety of sample sizes, consumer risks, reliability objectives, and Weibull shape parameters. All ratios presented in Appendix 6 have been rounded up to the nearest two digits. The following Tables, excerpted from Appendix 6, show how the appendix may be used.

SAMPLE SIZE = 30 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	99.83	65.69	46.35	34.51	26.8	21.52
.99	9.94	8.07	6.78	5.85	5.16	4.63
.98	4.95	4.28	3.79	3.42	3.14	2.91
.97	3.28	2.95	2.69	2.5	2.34	2.21
.96	2.45	2.26	2.11	1.99	1.9	1.82
.95	1.95	1.84	1.75	1.67	1.61	1.56
.94	1.62	1.55	1.5	1.45	1.41	1.38
.93	1.38	1.34	1.31	1.28	1.26	1.24
.92	1.2	1.18	1.17	1.15	1.14	1.13
.91	1.06	1.06	1.05	1.05	1.05	1.04
.9	.95	.96	.96	.96	.97	.97

Table 13

Zero Failure Criteria: $n = 30, \beta = .05$

SAMPLE SIZE = 30 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	76.73	51.72	37.23	28.19	22.21	18.06
.99	7.64	6.35	5.45	4.78	4.28	3.88
.98	3.8	3.37	3.05	2.8	2.6	2.44
.97	2.52	2.32	2.17	2.04	1.94	1.86
.96	1.89	1.78	1.7	1.63	1.57	1.53
.95	1.5	1.45	1.4	1.37	1.34	1.31
.94	1.25	1.22	1.2	1.19	1.17	1.16
.93	1.06	1.06	1.05	1.05	1.05	1.04
.92	.93	.93	.94	.94	.95	.95
.91	.82	.83	.85	.86	.87	.88
.9	.73	.75	.77	.79	.8	.81

Table 14
Zero Failure Criteria: $n = 30, \beta = .1$

SAMPLE SIZE = 40 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	57.55	39.82	29.29	22.59	18.08	14.91
.99	5.73	4.89	4.29	3.83	3.48	3.21
.98	2.85	2.6	2.4	2.24	2.12	2.01
.97	1.89	1.79	1.7	1.64	1.58	1.53
.96	1.42	1.37	1.34	1.31	1.28	1.26
.95	1.13	1.12	1.11	1.1	1.09	1.08
.94	.94	.94	.95	.95	.95	.96
.93	.8	.82	.83	.84	.85	.86
.92	.7	.72	.74	.76	.77	.79
.91	.62	.64	.67	.69	.71	.72
.9	.55	.58	.61	.63	.65	.67

Table 15
Zero Failure Criteria: $n = 40, \beta = .1$

The Figures in each column of Tables 13, 14, and 15 are the ratio T/t discussed previously. When using Appendix 6 in conjunction with another test, such as our previous H-108 example, always use the next lower sample size that is available in Appendix 6. Thus if the population has a constant or rising hazard rate, select Table 14, for a consumer risk of 0.1 and a sample size of 30 as quick cross check against the previous example. No failures by 75 hours (50 X 1.5 = 75) provides early evidence that the minimum reliability requirement has been met. The 1.5 was selected by reading across from the reliability requirement of 0.95 under the shape parameter of 1.0 (an exponential distribution). This time is longer than the precise calculation for n = 34 but it is conservative.

Comparing Table 13 and Table 14, it is apparent that increasing the consumer risk allows an earlier decision for a given sample size. Comparing Table 14 and Table 15, it is apparent that increasing the sample size allows an earlier decision for a given consumer risk. This is consistent with the discussion in earlier sections.

Development of Appendix 6 originates from the non-parametric question of what is the minimum reliability just prior to the first failure consistent with a given sample size and consumer risk. Time is introduced by asking when a certain failure model, selected by test data analysis or engineering judgment, meets both a predetermined reliability requirement and that reliability which is consistent with the sample size and consumer risk. In a sense, all truncated tests are non-parametric. There has not been a test to failure of a sufficiently large sample to precisely identify the failure distribution.

Perusal of Appendix 6 shows that as the shape parameter increases, the time at which the test may be terminated with zero failures more closely approaches the time specified for the reliability requirement. The ratio is approaching one. For items with rapidly increasing hazard rates, it

can be demonstrated that the equipment meets the minimum reliability requirement at test termination and also shows that many failures may be expected shortly thereafter. This is an element of risk not generally considered in reliability specifications. It argues strongly in favor of conducting any truncated test well beyond the expected operating times. The engineer may not want equipment that promises to fail shortly after the expected operating time. Certainly, how the expected operating time was determined should be reviewed.

As an example, suppose that in a test similar to the previous example from H-108, 34 items were tested with the first failure at 59 hours and the sixth failure at 100 hours. Computing an estimated Weibull shape parameter gives:

$$\text{Est } m = \frac{\ln\ln\left[\frac{1}{(1-6/35)}\right] - \ln\ln\left[\frac{1}{(1-1/35)}\right]}{\ln 100 - \ln 59}$$

$$= 3.54$$

This result may come from a Weibull population with a shape parameter of m = 1.5 or lower, but the probability of the true shape parameter being this low is quite small. The termination time as calculated in the previous section, of 166 hours is beyond the time of the sixth failure and thus the test would fail for m = 1.5. There is thus a temptation to use a higher shape parameter.

If the true shape parameter were 3.5, the test termination time would be 85 hours and the equipment would meet the minimum reliability requirement. However, the system's sensitivity to operating beyond 50 hours is quite high. To see why, note that there is a ten percent consumer's risk that the true reliability just prior to the first failure at 59 hours is as

low as:

$$\text{Est. } R = (.1)^{1/34} = .9345$$

Calculating the scale parameter consistent with a reliability of .9345 and a shape parameter of 3.5 gives:

$$R = e^{-(t/\theta)^m}$$

$$.9345 = e^{-(59/\theta)^{3.5}}$$

$$\theta = \frac{59}{\left[-\ln(.9345)\right]^{\frac{1}{3.5}}}$$

$$= 127 \text{ hours}$$

Computing the reliability at 50, 75, and 100 hours illustrates the nature of the problem.

$$R_{50} = e^{-(50/127)^{3.5}} = .9624$$

$$R_{75} = e^{-(75/127)^{3.5}} = .8536$$

$$R_{100} = e^{-(100/127)^{3.5}} = .6484$$

There is a marked degradation in relability should the equipment operate beyond the expected 50 hours.

There are three risks in accepting the equipment. First is the risk that the equipment may be operated beyond the expected 50 hours. Second is the risk that the true reliability is lower than expected due to the true population scale parameter being lower than expected (MTBF effect). Third is the risk that the true reliability is lower than expected due to the true shape parameter being lower than expected (hazard rate effect). If the time to first failure is well beyond the expected operating time, the risk in estimating too high of a shape parameter is substantially reduced. Otherwise, the engineer must carefully consider whether he can live with a substantial degradation of reliability shortly after his minimum acceptable reliability has been achieved. Certainly, he will want to test until there is more than one failure in order to provide some data as to how the hazard rate may vary with time.

XIII THE PROBABILITY RATIO SEQUENTIAL TEST

Prior discussion has focused upon four critical decisions that the test engineer must make in determining a demonstration test.

1. The reliability requirement
2. The desired confidence level (acceptable consumer's risk)
3. The sample size
4. The length of the test

Assuming that time is available, costs may be reduced by testing a few items for a longer time rather than many items for a short period. Analysis of failure data may allow even further reductions in test time.

Time or failure terminated tests have been replaced in practice by the Probability Ratio Sequential Test (PRST) developed during World War II by Wald. Strictly speaking the PRST does not validate a specific reliability requirement. It does not choose, to a pre-determined risk, between the hypothesis that a given item meets a pre-determined reliability requirement and the alternate hypothesis, that it does not. The PRST chooses between two different reliability requirements which are usually quite far apart. The test proceeds, testing item by item, until the cumulative test time to failure exceeds an acceptance criteria or falls below a rejection criteria. Acceptance means that the test has determined to a low consumer risk, β , that the component is more reliable than the minimum acceptable reliability. Rejection means that the test has determined to a low producer risk, α , that the component is less reliable than the desired higher reliability.

The PRST does not determine an estimate of the true reliability. Analysis far beyond the scope of this report shows that the PRST is the least time consuming method to discriminate between the two reliabilities that define the test. This assumes that the true reliability exceeds or

equals either extreme. This promised time saving accounts for the wide usage of the PRST.

Appendix 3 provides formulas for a variety of PRST accept and reject criteria. These formulas are all linear for the statistic being measured. Simplified versions of the formulas for the three failure distributions used in this handbook are as follows:

1. Binomial

$$S = F \cdot \frac{\ln\left(\frac{1-R_L}{1-R_U}\right)}{\ln\left(\frac{R_U}{R_L}\right)} \pm \frac{\ln\left(\frac{\beta}{1-\alpha}\right)}{\ln\left(\frac{R_U}{R_L}\right)}$$

Where

R_U = desired (design) reliability

R_L = minimum acceptable reliability

α = producers risk - probability of rejecting a lot whose reliability is equal to or greater than R_U

β = consumers risk - probability of accepting a lot whose reliability is equal to or less than R_L

S = number of successes

F = number of failures

Note: Consumer and producer risks must be equal for the above formula to be true ($\beta = \alpha$). The accept line will be the larger value of S as computed above.

2. Exponential

$$\sum\left(\frac{TTF}{\theta_1}\right) = \frac{F(d)\left[\ln(d)\right]}{d-1} \pm \frac{d\left[\ln\left(\frac{\beta}{1-\alpha}\right)\right]}{d-1}$$

Where:

d = θ_0/θ_1, the discrimination ratio ($\theta_0 > \theta_1$)

θ_0 = Desired (design) MTBF

θ_1 = Lowest acceptable MTBF

α = Producer's risk - probability of rejecting a lot whose MTBF is equal to or better than θ_0

β = Consumer's risk - probability of accepting a lot whose MTBF is equal to or less than θ_1

F = Number of failures

TTF = Time to failure

Note: The above formula assumes $\beta = \alpha$. The accept line will be the larger $\sum\left(\frac{TTF}{\theta_1}\right)$ as computed above.

3. Weibull

$$\sum\left(\frac{TTF}{\theta_1}\right)^m = \frac{F(d^m)(m)\left[\ln(d)\right]}{d^m-1} \pm \frac{d^m\left[\ln\left(\frac{\beta}{1-\alpha}\right)\right]}{d^m-1}$$

Where:

d = θ_o/θ_1, the discrimination ratio ($\theta_o > \theta_1$)

θ_o = Desired (design) scale parameter

θ_1 = Lowest acceptable scale parameter

α = Producer's risk - probability of rejecting a lot whose scale parameter is equal to or better than θ_o

β = Consumer's risk - probability of accepting a lot whose scale parameter is equal to or less than θ_1

F = Number of failures

TTF = Time to failure

m = Weibull Shape Parameter

Note: The above formula assumes $\beta = \alpha$. The accept line will be the larger $\sum \left(\frac{TTF}{\theta_1}\right)^m$ as computed above.

For an initial look at the PRST, consider the binomial model which might be used to evaluate a one-shot device, such as a squid. Figure 17 presents a plot of the accept and reject criteria for the preceding binomial PRST formula for the following test.

R_U = .95

R_L = .90

$\alpha = \beta = 0.20$

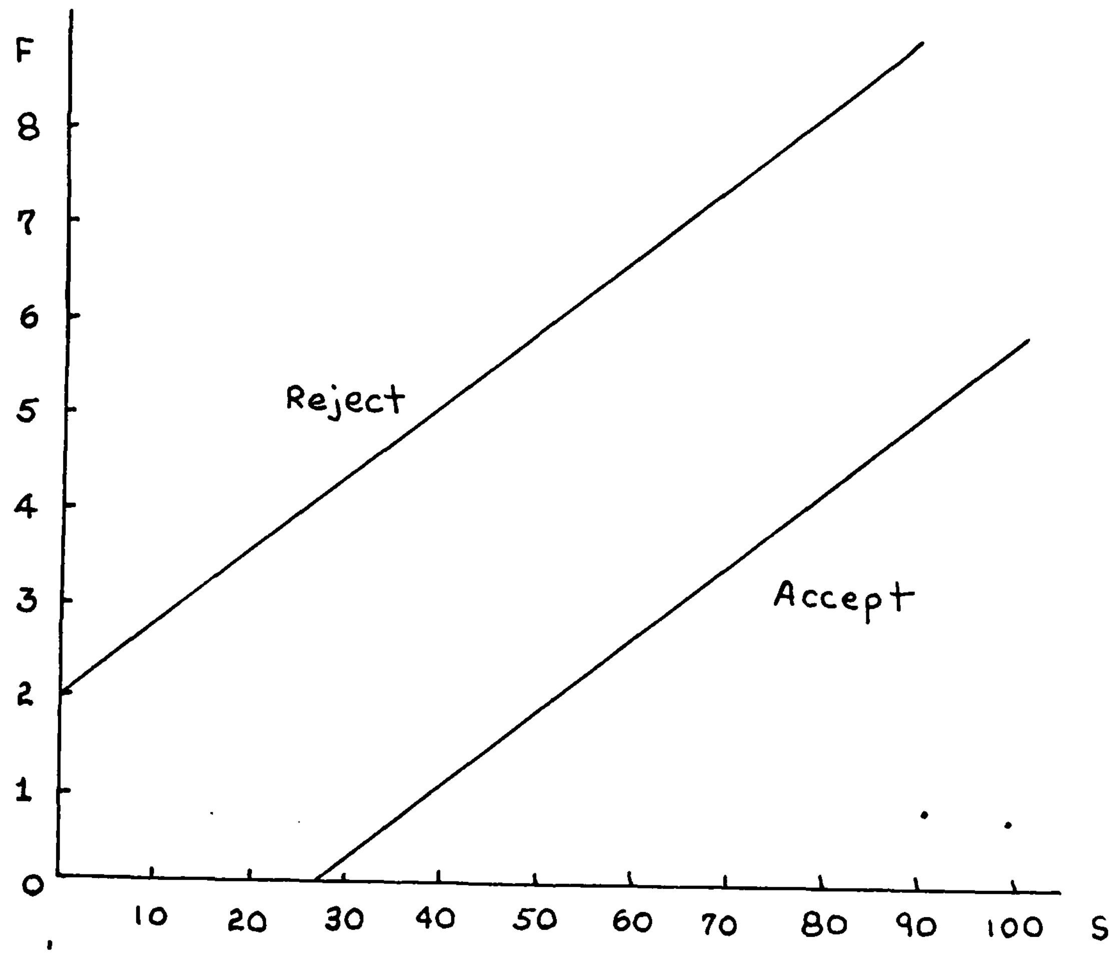


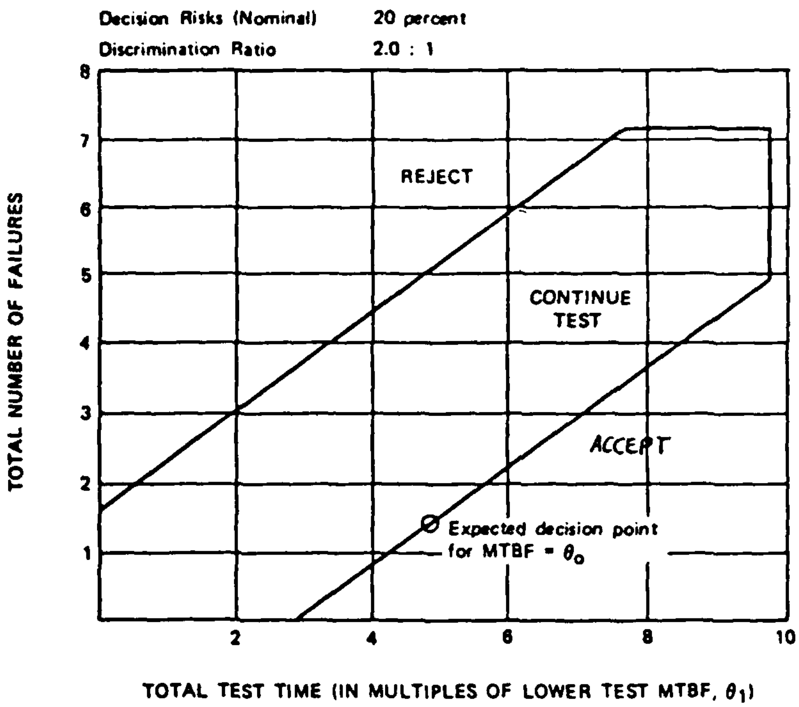
Figure 17
PRST for a Binomial Failure Model

The procedure is to begin testing the lot by selecting at random one item at a time and performing the test. If the cumulative failures and successes are plotted per Figure 17 the plot should eventually cross the accept or the reject line. If the first twenty six items tested passed, the lot would be accepted. A large number of other success/failure combinations exist. In fact, the test may well proceed indefinitely, if the true reliability lies between the desired reliability and the minimum

acceptable reliability. To prevent this, the engineer can establish accept/reject criteria based upon the binomial distribution for a selected maximum sample size and terminate the test, using the binomial criteria when sufficient testing has been accomplished. Using the PRST at the beginning will save considerable test time through early discrimination whenever the true reliability is unusually high or low compared to the average of the upper and lower acceptable reliabilities.

The possibility of an early test termination is so advantageous that MIL-STD-781C, which establishes PRST criteria for exponential failure distributions, is probably the most widely used Air Force specification for reliability demonstration testing. As an example, Figure 18, from MIL-STD-781C, shows accept/reject criteria for test plan IV C.

For reasons of historical development, the various test plans in MIL-STD-781C do not have exactly the nominal decision risks stated at the top of each acceptance plan. The accept/reject criteria as determined by the preceding exponential PRST formula (for $\alpha = \beta = .20$ and a discrimination ratio of $d = 2.0$) is shown in Figure 19, also from MIL-STD-781C. These criteria are slightly different from Figure 18.



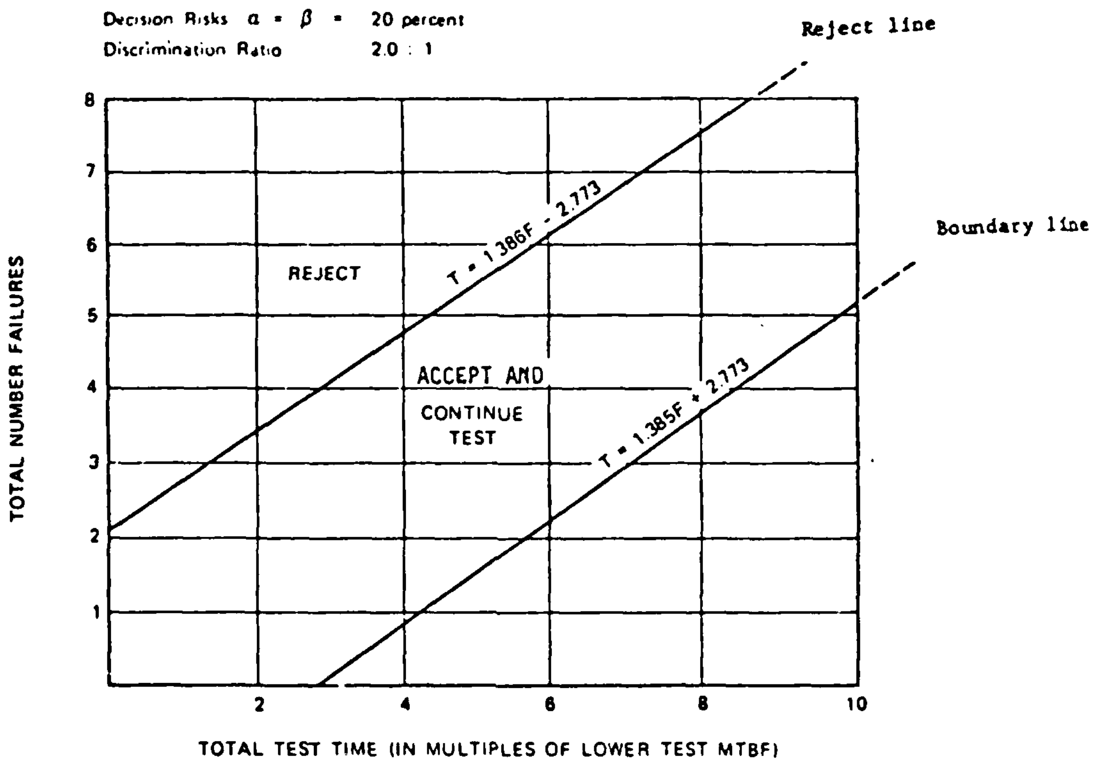
Total Test Time*

Number of Failures	Reject (Equal or less)	Accept (Equal or more)
0	N/A	2.80
1	N/A	4.18
2	.70	5.58
3	2.08	6.96
4	3.46	8.34
5	4.86	9.74
6	6.24	9.74
7	7.62	9.74
8	9.74	N/A

* Total test time is total unit hours of equipment on time and is expressed in multiples of the lower test MTBF. Refer to 4.5.2.4 for minimum test time per equipment.

Figure 18

MIL-STD-781C Test Plan IV C



Total Test Time*

Number of Failures	Reject (Equal or less)	Boundary Line
0	N/A	2.77
1	N/A	4.16
2	N/A	5.55
3	1.39	6.93
4	2.77	8.32
5	4.16	9.70
6	5.54	11.09
7	6.93	12.48
8	8.32	13.86

*Total test time is total unit hours of equipment on time and is expressed in multiples of the lower test MTBF. Refer to 4.5.4.1 for minimum test time per equipment.

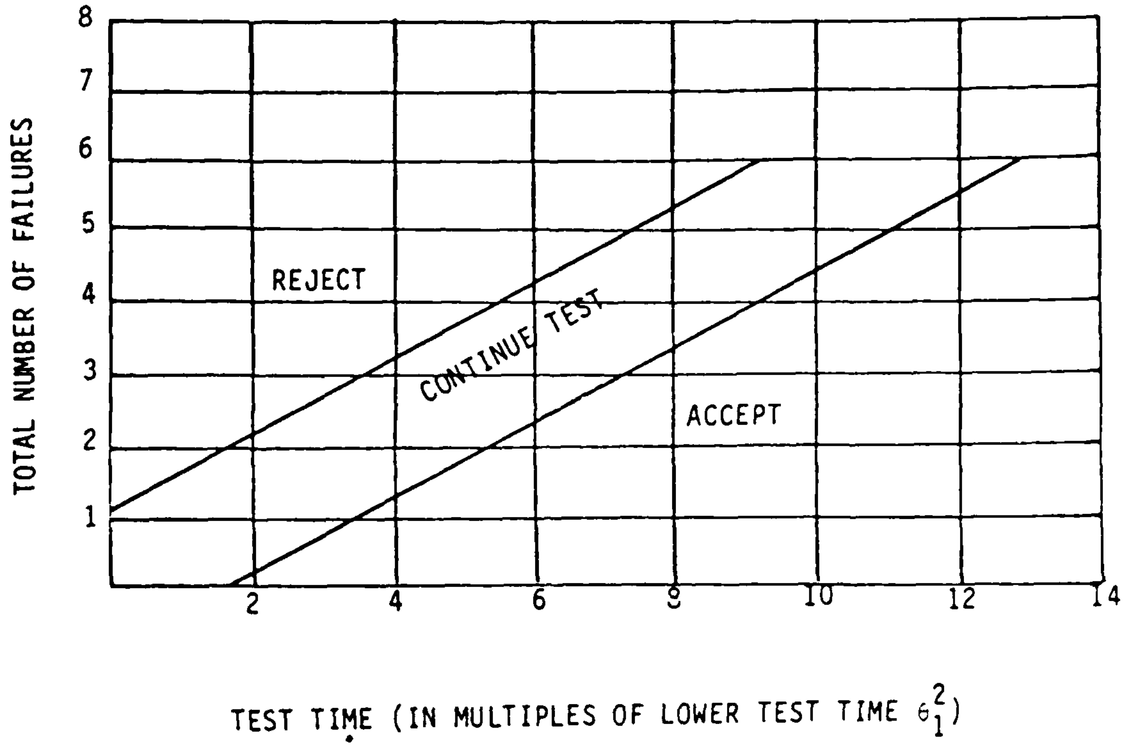
Figure 19

PRST for an Exponential Failure Model

Weibull Distribution

Decision Risks (Nominal) 20 percent

Discrimination Ratio 2.0 : 1



NUMBER OF FAILURES	REJECT (EQUAL OR LESS)	ACCEPT (EQUAL OR MORE)
0	N/A	1.85
1	N/A	3.69
2	1.85	5.54
3	3.69	7.39
4	5.52	9.24
5	7.39	11.09
6	9.24	12.96

Figure 20

PRST for a Weibull Failure Model

Figure 20, from reference 8, presents the accept/reject criteria for a Weibull distribution with:

$\alpha = \beta =$	.20	decision risks
m =	2	shape parameter
d =	2.0	discrimination ratio

This figure derives directly from the preceding Weibull P?ST formula.

XIV The Exponential PRST vs the Weibull PRST

In the case of truncated tests, it was possible to derive directly the reduction in test termination time to be allowed, provided that the engineer could ascertain that he was in fact testing a Weibull population as compared to an exponential population (Figure 11 and Appendix 2). The PRST does not lend itself to a similar derivation. An understanding of the potential savings that may be gained can be ascertained by examining figure 21.

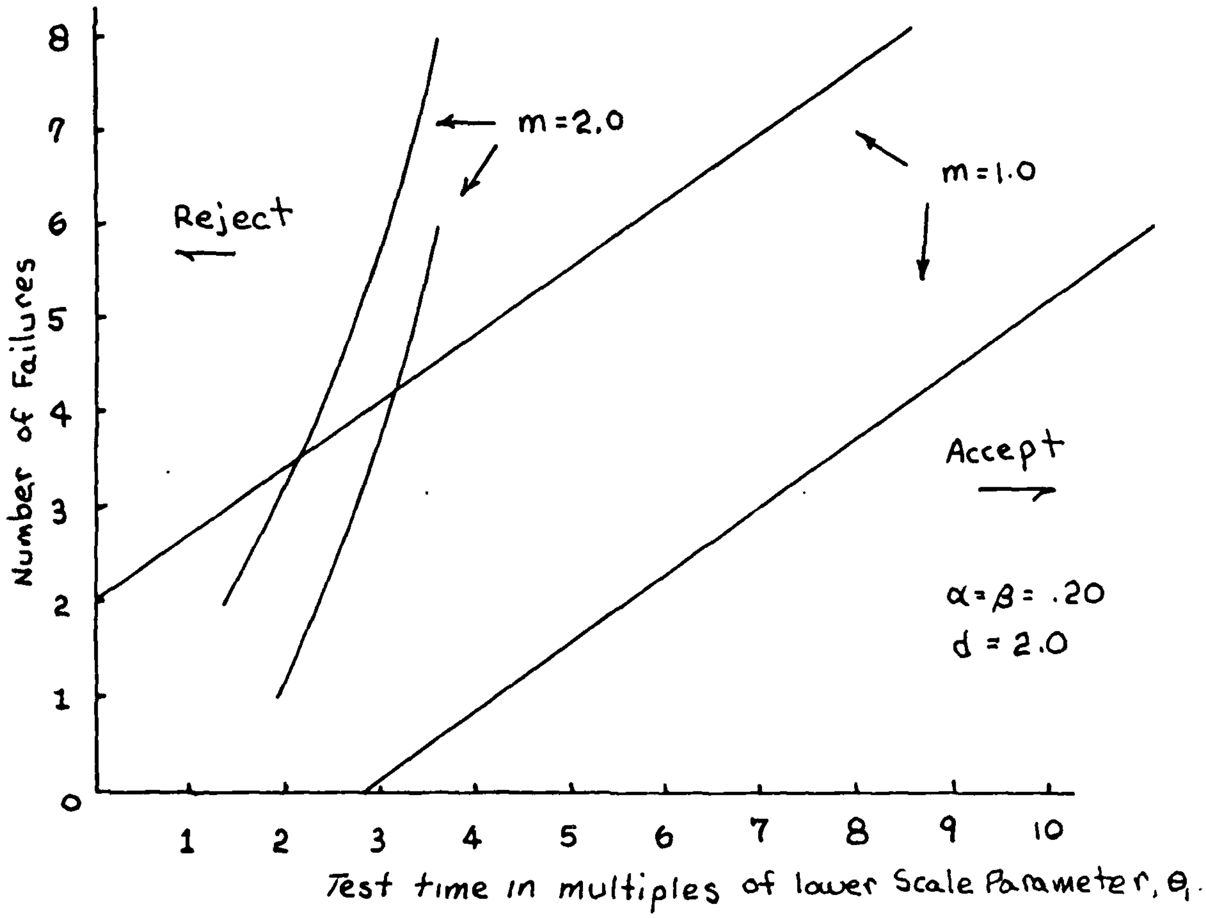


Figure 21
Exponential Vs Weibull PRST

Figure 21 plots the exponential and Weibull distributions of the previous section (figure 19 and figure 20) in terms of the lower scale parameter, Θ_1 , for both distributions. Note that Θ_1 is the familiar MTBF in the case of the exponential distribution.

It is apparent that, if the lot comes from a Weibull population, it is most likely to be rejected when using an exponential PRST test such as MIL-STD-781C unless the scale parameter, Θ , of the Weibull population should be considerably in excess of the design MTBF, θ_0 , for the exponential. This need not reject the lot if the Weibull distribution itself is acceptable. Previous discussion has shown that, if both a Weibull and an exponential distribution have the same reliability at a given time, the Weibull will have greater reliability at shorter times and less reliability at longer times. (Note: The preceding statement assumes a shape parameter greater than one.) The engineer will want to ensure that the lowest acceptable Weibull scale parameter is well in excess of the expected equipment operating time.

Not knowing the true nature of the population's failure distribution, selection of an exponential PRST is conservative in the same sense as discussed previously. Equipment passing the test will be of equal or better reliability to the assumed exponential within the operating regime where high reliability is desired. This assumes that the true hazard rate is increasing with time. This assumption is likely to be true in the case of non-electronic components, where wear-out failures argue for a hazard rate that increases with time. The serious question occurs upon failure of the test. Could the equipment meet reliability requirements within the operational time period even though it failed the test? An initial clue is provided if all failures occur well beyond the time for which the equipment is expected to operate. This result invites further analysis to determine the nature of the failure distribution.

Consistent with earlier discussion on truncated tests, the procedure advised is to plot the failure points on Weibull graph paper and see how close the plot conforms to a straight line. An estimate of the shape parameter can be derived from the first and last failure point according to the formula:

$$m = \frac{\ln\ln\left[\frac{1}{R(t_2)}\right] - \ln\ln\left[\frac{1}{R(t_1)}\right]}{\ln(t_2) - \ln(t_1)}$$

Where:

t_2 = last failure time

t_1 = first failure time

$R_{(t)} = 1 - \frac{r}{n+1}$

n = number of failures (R in Table 16)

Some confidence in the results obtained from the above formula can be provided by simulation studies. Table 16 shows the results of the following simulation study. A sample of n random numbers between .00001 and .99999 were computer generated and converted to failure times, assuming that each number represented the reliability of a component which failed exponentially with a MTBF of unity. This was done for one thousand iterations with the time of the first and last failures, along with the number of assumed failures, n, applied to the above formula. Note: Symbol R, for rejection number, in Table 16 equals n in the above formula.

The figures within Table 16 estimate the proportion of computed shape parameters from a population with a shape parameter of one which are expected to fall below the higher estimate shown at the top of each column. This proportion is, thus, a measure of the engineer's confidence, having computed a given higher value for the shape parameter, that the true shape parameter is greater than one.

TOTAL TRIALS SIMULATED 1000
MAXIMUM NUMBER OF FAILURES 10
POPULATION SHAPE PARAMETER 1

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.661	.745	.791	.82	.848	.874
3	.782	.857	.916	.939	.954	.965
4	.85	.914	.954	.977	.989	.991
5	.875	.946	.975	.983	.994	.997
6	.911	.976	.988	.995	.997	.998
7	.904	.971	.987	.994	.997	.999
8	.94	.988	.993	.999	.999	1
9	.934	.99	.999	1	1	1
10	.95	.989	.998	1	1	1

Table 16

Proportion of Shape Parameter Estimates
Below Specified Levels

In order to provide additional criteria, similar simulations were performed with the true shape parameter varying from 1.0 to 1.5. The results are shown in Appendix 5. Again, the figures presented are a measure of confidence, having computed a given higher value for the shape parameter, that the true population shape parameter is larger than the population shape parameter used for each simulation (listed directly under the maximum number of failures). The tables of Appendix 5 differ from Appendix 4 because we are now simulating a complete test to failure of all of the two to ten items tested. In Appendix 4 we were simulating the ten earliest failures for sample sizes that varied from n=10 to n=40. Like Appendix 4, the low number of simulations provides only an approximate estimate of the confidence level.

While the usage of Appendix 5 may provide strong indication that failures are occurring from a Weibull population, or at least some distribution with an increasing hazard rate, the PRST is not the proper approach to making the accept-reject decision. The sample size is usually too small

to determine the exact nature of the population failure distribution. The proper procedure is to expand the test to a reasonably large sample size and then conduct goodness of fit tests to gain confidence in the nature of the failure distribution. A reasonable estimate can then be made regarding the probability of having met the minimum acceptable reliability requirement. Reference 15 section 3.0 as well as standard statistical textbooks illustrate appropriate techniques for analyzing samples that have been tested to failure. Contrary to truncated tests, sufficient data will exist in tests to failure to enable using a test for two parameter distributions, such as the normal distribution. For the normal distribution, considerable theory exists relating to small sample data and its relationship to the population parameters which define the distribution. This is less true for other distributions. If time is lacking, you may want to use a truncated test as discussed earlier.

XV SUMMARY

The purpose of this handbook has been to explore considerations necessary for testing the reliability of non-electronic parts, where the hazard rate is expected to increase with time. The Weibull distribution, because of its flexibility, was chosen as a mathematical model for estimating reliability at various times. Computer simulations were performed to see the extent to which estimates of the Weibull shape parameter may vary when estimated from a small sample. Without repeating the mathematics, let's review in graphical format what was discussed.

Reliability is a proportional concept which is the ratio of components that can successfully operate for a given time to all similar components. Reliability can never be less than zero nor more than one. Subtracting the reliability from one gives its complement, the probability of failure. Reliability and the probability of failure always add up to one.

The engineer never knows a component's precise reliability. He can only estimate a range within which the true reliability is expected to exist to some predetermined confidence. The concept of confidence is also a proportional concept which expresses the proportion of total tests of many similar tests that may be performed with results equal to or less than the result which was obtained, should the true reliability be as low as the lower confidence limit. Since the engineer is interested only in how low the reliability may be, the highest possible value, represented by the upper confidence limit, is generally not of interest.

The typical design requires quite high reliability during a reasonable equipment operating time. A sample is then tested to provide confidence that the design has met its objective. This test involves operating the component for a specified time, which may be until failure occurs. A time terminated test is called a truncated test. A failure terminated

test may also be truncated, unless all items in the sample are tested to failure. Truncated tests are essentially binomial tests where the time of termination or number of failures in a given time, is defined by the nature of the population's failure distribution.

Analysis shows that, for highly reliable components, extremely large samples are required to demonstrate the reliability whenever the test is terminated at the normal operating time. This is because of the very low probability of failure desired at this time. Demonstrating a much lower reliability at some later time can be accomplished with a smaller sample. In general, the sample size reduces proportionately as the probability of failure, expected to occur by test termination, increases. One way to reduce the cost of testing, when set up and component costs are high, is to extend the test time well beyond normal equipment operating time.

How long a truncated test must proceed depends upon some knowledge or reasonable assumptions, as to how the components fail with time. For a reasonably small sample, the test must proceed until perhaps twenty percent of the components may be expected to fail. Then, if only a few fail, the engineer may be quite confident that his components met the reliability objective.

Before testing, the engineer has no knowledge of when the components may fail during operation. Within the Air Force, the most widely used assumption is that components fail exponentially. Indeed, electronic components do tend to fail exponentially. If the number of failures that occur during small intervals of time is plotted, the graph of an exponential failure distribution looks like Figure 22.

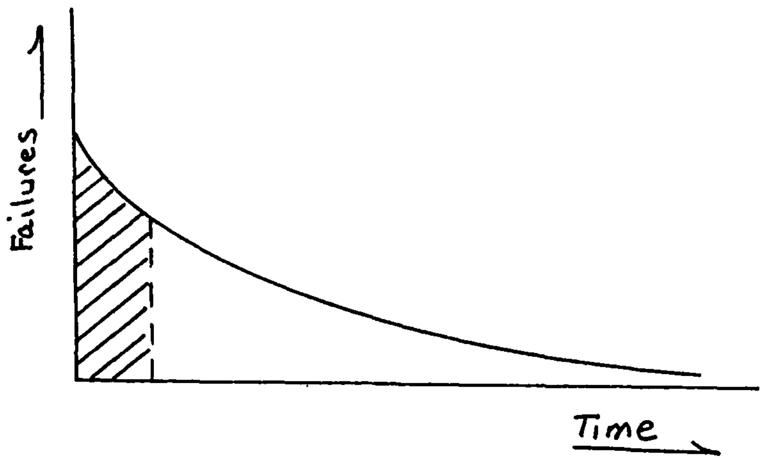


Figure 22
Exponential Failure Distribution

Dividing the number of failures that have occurred (cross hatched area) by the total area under the graph gives the probability of failure at that time. The exponential distribution is unique in that, even after many failures, taking the number of failures in some time increment and dividing by the remaining number of failures (area remaining under the curve) provides the same probability of failure for the remaining components as would be expected to occur in the same increment at an earlier (or later) time. The exponential distribution thus has a constant hazard rate.

Contrary to electronic components, non-electronic components are likely to have a hazard rate that increases with time since their failure mode (method of failure) results from accumulated wear and operational stress. A similar graph for non-electronic components may look like figure 23.

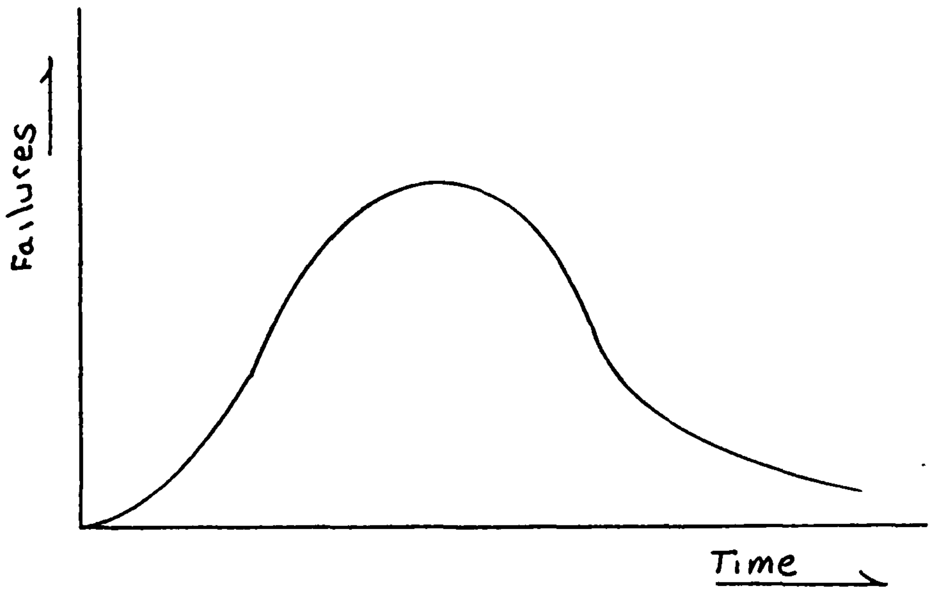


Figure 23
Failure Distribution - Mechanical Components

In order to compare various failure distributions a different type of graph is necessary. This is a plot of how the probability of failure increases with time. For a given reliability requirement (minimum reliability for a given operating time) such a plot may look like Figure 24.

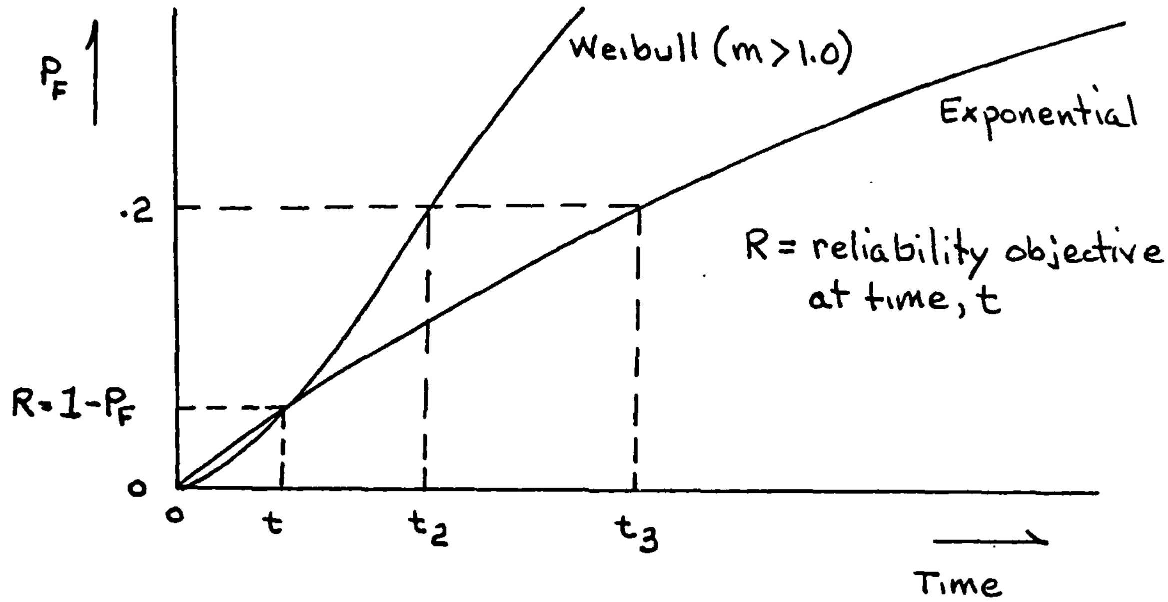


Figure 24
Cumulative Failure Distributions

Observe that, even though both curves have the same reliability, R, at time, t, the curve with the increasing hazard rate will reach a probability of failure, P_F, of 0.2 much sooner (t_2) than the curve with the constant hazard rate (t_3).

A truncated test does not estimate the true reliability at time, t. The test simply provides confidence that the reliability at time, t, is consistent with the test termination time, t_2 or t_3, at which $P_F = 0.2$. Testing until $P_F = 0.2$ is a decision necessary to enable a reasonably small sample size. Confidence that the population distribution hazard rate is increasing with time according to a Weibull process, where $m > 1.0$, allows shortening of the test termination time from t_3 to t_2 without violating the restriction that $P_F = 0.2$. The restriction that $P_F = 0.2$

is necessary to maintain the sample size.

Throughout this report, the Weibull distribution was extensively explored because of the distribution's ability to model a wide variety of conditions wherein hazard rates vary with time. The exponential distribution is a special case of the Weibull, where the shape parameter is equal to unity. For any specified reliability at time, t, a Weibull distribution with a greater shape parameter will have increased reliability at time less than t and will have reduced reliability at time greater than t, as compared to any Weibull with a lower shape parameter. Since truncated tests are terminated at a specified probability of failure, a Weibull with a greater shape parameter, which also passes the test, will meet the reliability objective.

It is common to test non-electronic components as though they failed exponentially. The problem arises when the component fails the exponential test. If sufficient data is provided from the failures to reasonably estimate a Weibull distribution that models the early failures, it may be possible to show that the reliability objective was met. This is done by counting failures at a lesser time ($t_2 < t_3$) consistent with the additional data provided from the failures. This approach is indicated whenever most of the failures occur late in the test.

A preponderance of late test failures may result from the parent population being in fact a Weibull distribution with a larger shape parameter. However, the results could also proceed by chance from random variation in any small sample even though the parent population is in fact exponential. To assess the probability of such sampling errors, simulation studies were performed to estimate the proportion of small samples which result in test statistics (estimates of the shape parameter) with values in excess of the true parent population. These studies provided a basis for confidence in reducing the test termination time should

analysis of the failure data indicate the advisability of doing so.

A more popular method for reliability demonstration testing is the Probability Ratio Sequential Test (PRST). This is a test to failure of components which are selected one by one, at random, until the accumulated time to failure either exceeds an acceptance limit or falls below a rejection limit. Acceptance limits are established based upon a known or an assumed failure distribution and must be consistent with an acceptable versus an unacceptable reliability objective. The PRST has the advantage of quickly discriminating between the alternatives, provided that the true reliability approaches either extreme. If the true reliability is in the middle, the test may continue for too large of a sample. To prevent this, the test is frequently terminated at some reasonable small sample size.

Like time truncated tests, the PRST presently in use is based upon the exponential failure distribution model. Unfortunately, the increasing hazard rate associated with non-electronic components makes it quite likely that an exponential PRST will result in failure when applied to non-electronic components which may yet satisfy the reliability objective. Also, the reject decision is likely to be made when the total number tested is quite small. The small sample may be too limited to adequately determine the nature of the parent population. When confronted with this problem, the engineer must expand the sample size and apply an alternate test. This alternate may be a time truncated test. It may also be a test to failure of a sample large enough to analyze the nature of the failure distribution.

APPENDIX 1

CUMULATIVE CHI-SQUARE DISTRIBUTION

Source: Reference 5

$$p = P\left[\chi^2 \leq \chi^2_p\right]$$

	p										p										
ν	0.0005	0.001	0.005	0.01	0.025	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.975	0.99	0.995	0.999	0.9995
1	$0.0^{6}393$	$0.0^{5}157$	$0.0^{4}393$	$0.0^{3}157$	$0.0^{3}982$	$0.0^{2}393$	0.0158	0.0642	0.148	0.275	0.455	0.708	1.07	1.64	2.71	3.84	5.02	6.63	7.88	10.8	12.1
2	$0.0^{3}100$	$0.0^{2}200$	0.0100	0.0201	0.0506	0.103	0.211	0.446	0.713	1.02	1.39	1.83	2.41	3.22	4.61	5.99	7.38	9.21	10.6	13.8	15.2
3	0.0153	0.0243	0.0717	0.115	0.216	0.352	0.584	1.00	1.42	1.87	2.37	2.95	3.67	4.64	6.25	7.81	9.35	11.3	12.8	16.3	17.7
4	0.0639	0.0908	0.207	0.297	0.484	0.711	1.06	1.65	2.19	2.75	3.36	4.04	4.88	5.99	7.78	9.49	11.1	13.3	14.9	18.5	20.0
5	0.158	0.210	0.412	0.554	0.831	1.15	1.61	2.34	3.00	3.66	4.35	5.13	6.06	7.29	9.24	11.1	12.8	15.1	16.7	20.5	22.1
6	0.299	0.381	0.676	0.872	1.24	1.64	2.20	3.07	3.83	4.57	5.35	6.21	7.23	8.56	10.6	12.6	14.4	16.8	18.5	22.5	24.1
7	0.485	0.598	0.989	1.24	1.69	2.17	2.83	3.82	4.67	5.49	6.35	7.28	8.38	9.80	12.0	14.1	16.0	18.5	20.3	24.3	26.0
8	0.710	0.857	1.34	1.65	2.18	2.73	3.49	4.59	5.53	6.42	7.34	8.35	9.52	11.0	13.4	15.5	17.5	20.1	22.0	26.1	27.9
9	0.972	1.15	1.73	2.09	2.70	3.33	4.17	5.38	6.39	7.36	8.34	9.41	10.7	12.2	14.7	16.9	19.0	21.7	23.6	27.9	29.7
10	1.26	1.48	2.16	2.56	3.25	3.94	4.87	6.18	7.27	8.30	9.34	10.5	11.8	13.4	16.0	18.3	20.5	23.2	25.2	29.6	31.4
11	1.59	1.83	2.60	3.05	3.82	4.57	5.58	6.99	8.15	9.24	10.3	11.5	12.9	14.6	17.3	19.7	21.9	24.7	26.8	31.3	33.1
12	1.93	2.21	3.07	3.57	4.40	5.23	6.30	7.81	9.03	10.2	11.3	12.6	14.0	15.8	18.5	21.0	23.3	26.2	28.3	32.9	34.8
13	2.31	2.62	3.57	4.11	5.01	5.89	7.04	8.63	9.93	11.1	12.3	13.6	15.1	17.0	19.8	22.4	24.7	27.7	29.8	34.5	36.5
14	2.70	3.04	4.07	4.66	5.63	6.57	7.79	9.47	10.8	12.1	13.3	14.7	16.2	18.2	21.1	23.7	26.1	29.1	31.3	36.1	38.1
15	3.11	3.48	4.60	5.23	6.26	7.26	8.55	10.3	11.7	13.0	14.3	15.7	17.3	19.3	22.3	25.0	27.5	30.6	32.8	37.7	39.7
16	3.54	3.94	5.14	5.81	6.91	7.96	9.31	11.2	12.6	14.0	15.3	16.8	18.4	20.5	23.5	26.3	28.8	32.0	34.3	39.3	41.3
17	3.98	4.42	5.70	6.41	7.56	8.67	10.1	12.0	13.5	14.9	16.3	17.8	19.5	21.6	24.8	27.6	30.2	33.4	35.7	40.8	42.9
18	4.44	4.90	6.26	7.01	8.23	9.39	10.9	12.9	14.4	15.9	17.3	18.9	20.6	22.8	26.0	28.9	31.5	34.8	37.2	42.3	44.4
19	4.91	5.41	6.84	7.63	8.91	10.1	11.7	13.7	15.4	16.9	18.3	19.9	21.7	23.9	27.2	30.1	32.9	36.2	38.6	43.8	46.0
20	5.40	5.92	7.43	8.26	9.59	10.9	12.4	14.6	16.3	17.8	19.3	21.0	22.8	25.0	28.4	31.4	34.2	37.6	40.0	45.3	47.5
21	5.90	6.45	8.03	8.90	10.3	11.6	13.2	15.4	17.2	18.8	20.3	22.0	23.9	26.2	29.6	32.7	35.5	38.9	41.4	46.8	49.0
22	6.40	6.98	8.64	9.54	11.0	12.3	14.0	16.3	18.1	19.7	21.3	23.0	24.9	27.3	30.8	33.9	36.8	40.3	42.8	48.3	50.5
23	6.92	7.53	9.26	10.2	11.7	13.1	14.8	17.2	19.0	20.7	22.3	24.1	26.0	28.4	32.0	35.2	38.1	41.6	44.2	49.7	52.0
24	7.45	8.08	9.89	10.9	12.4	13.8	15.7	18.1	19.9	21.7	23.3	25.1	27.1	29.6	33.2	36.4	39.4	43.0	45.6	51.2	53.5
25	7.99	8.65	10.5	11.5	13.1	14.6	16.5	18.9	20.9	22.6	24.3	26.1	28.2	30.7	34.4	37.7	40.6	44.3	46.9	52.6	54.9
26	8.54	9.22	11.2	12.2	13.8	15.4	17.3	19.8	21.8	23.6	25.3	27.2	29.2	31.8	35.6	38.9	41.9	45.6	48.3	54.1	56.4
27	9.09	9.80	11.8	12.9	14.6	16.2	18.1	20.7	22.7	24.5	26.3	28.2	30.3	32.9	36.7	40.1	43.2	47.0	49.6	55.5	57.9
28	9.66	10.4	12.5	13.6	15.3	16.9	18.9	21.6	23.6	25.5	27.3	29.2	31.4	34.0	37.9	41.3	44.5	48.3	51.0	56.9	59.3
29	10.2	11.0	13.1	14.3	16.0	17.7	19.8	22.5	24.6	26.5	28.3	30.3	32.5	35.1	39.1	42.6	45.7	49.6	52.3	58.3	60.7
30	10.8	11.6	13.8	15.0	16.8	18.5	20.6	23.4	25.5	27.4	29.3	31.3	33.5	36.3	40.3	43.8	47.0	50.9	53.7	59.7	62.2
31	11.4	12.2	14.5	15.7	17.5	19.3	21.4	24.3	26.4	28.4	30.3	32.3	34.6	37.4	41.4	45.0	48.2	52.2	55.0	61.1	63.6
32	12.0	12.8	15.1	16.4	18.3	20.1	22.3	25.1	27.4	29.4	31.3	33.4	35.7	38.5	42.6	46.2	49.5	53.5	56.3	62.5	65.0
33	12.6	13.4	15.8	17.1	19.0	20.9	23.1	26.0	28.3	30.3	32.3	34.4	36.7	39.6	43.7	47.4	50.7	54.8	57.6	63.9	66.4
34	13.2	14.1	16.5	17.8	19.8	21.7	24.0	26.9	29.2	31.3	33.3	35.4	37.8	40.7	44.9	48.6	52.0	56.1	59.0	65.2	67.8
35	13.8	14.7	17.2	18.5	20.6	22.5	24.8	27.8	30.2	32.3	34.3	36.5	38.9	41.8	46.1	49.8	53.2	57.3	60.3	66.6	69.2
36	14.4	15.3	17.9	19.2	21.3	23.3	25.6	28.7	31.1	33.3	35.3	37.5	39.9	42.9	47.2	51.0	54.4	58.6	61.6	68.0	70.6
37	15.0	16.0	18.6	20.0	22.1	24.1	26.5	29.6	32.1	34.2	36.3	38.5	41.0	44.0	48.4	52.2	55.7	59.9	62.9	69.3	72.0
38	15.6	16.6	19.3	20.7	22.9	24.9	27.3	30.5	33.0	35.2	37.3	39.6	42.0	45.1	49.5	53.4	56.9	61.2	64.2	70.7	73.4
39	16.3	17.3	20.0	21.4	23.7	25.7	28.2	31.4	33.9	36.2	38.3	40.6	43.1	46.2	50.7	54.6	58.1	62.4	65.5	72.1	74.7
40	16.9	17.9	20.7	22.2	24.4	26.5	29.1	32.3	34.9	37.1	39.3	41.6	44.2	47.3	51.8	55.8	59.3	63.7	66.8	73.4	76.1
41	17.5	18.6	21.4	22.9	25.2	27.3	29.9	33.3	35.8	38.1	40.3	42.7	45.2	48.4	52.9	56.9	60.6	65.0	68.1	74.7	77.3
42	18.2	19.2	22.1	23.7	26.0	28.1	30.8	34.2	36.8	39.1	41.3	43.7	46.3	49.5	54.1	58.1	61.8	66.2	69.3	76.1	78.8
43	18.8	19.9	22.9	24.4	26.8	29.0	31.6	35.1	37.7	40.0	42.3	44.7	47.3	50.5	55.2	59.3	63.0	67.5	70.6	77.4	80.2
44	19.5	20.6	23.6	25.1	27.6	29.8	32.5	36.0	38.6	41.0	43.3	45.7	48.4	51.6	56.4	60.5	64.2	68.7	71.9	78.7	81.5
45	20.1	21.3	24.3	25.9	28.4	30.6	33.4	36.9	39.6	42.0	44.3	46.8	49.5	52.7	57.5	61.7	65.4	70.0	73.2	80.1	82.9
46	20.8	21.9	25.0	26.7	29.2	31.4	34.2	37.8	40.5	43.0	45.3	47.8	50.5	53.8	58.6	62.8	66.6	71.2	74.4	81.4	84.2
47	21.5	22.6	25.8	27.4	30.0	32.3	35.1	38.7	41.5	43.9	46.3	48.8	51.6	54.9	59.8	64.0	67.8	72.4	75.7	82.7	85.6
48	22.1	23.3	26.5	28.2	30.8	33.1	35.9	39.6	42.4	44.9	47.3	49.8	52.6	56.0	60.9	65.2	69.0	73.7	77.0	84.0	86.9
49	22.8	24.0	27.2	28.9	31.6	33.9	36.8	40.5	43.4	45.9	48.3	50.9	53.7	57.1	62.0	66.3	70.2	74.9	78.2	85.4	88.2
50	23.5	24.7	28.0	29.7	32.4	34.8	37.7	41.4	44.3	46.9	49.3	51.9	54.7	58.2	63.2	67.5	71.4	76.2	79.5	86.7	89.6

n	p: 0.0005	0.001	0.005	0.01	0.025	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.975	0.99	0.995	0.999	0.9995
51	24.1	25.4	28.7	30.5	33.2	35.6	38.6	42.4	45.3	47.8	50.3	52.9	55.8	59.2	64.3	68.7	72.6	77.4	80.7	88.0	90.9
52	24.8	26.1	29.5	31.2	34.0	36.4	39.4	43.3	46.2	48.8	51.3	53.9	56.8	60.3	65.4	69.8	73.8	78.6	82.0	89.3	92.2
53	25.5	26.8	30.2	32.0	34.8	37.3	40.3	44.2	47.2	49.8	52.3	55.0	57.9	61.4	66.5	71.0	75.0	79.8	83.3	90.6	93.5
54	26.2	27.5	31.0	32.8	35.6	38.1	41.2	45.1	48.1	50.8	53.3	56.0	58.9	62.5	67.7	72.2	76.2	81.1	84.5	91.9	94.8
55	26.9	28.2	31.7	33.6	36.4	39.0	42.1	46.0	49.1	51.7	54.3	57.0	60.0	63.6	68.8	73.3	77.4	82.3	85.7	93.2	96.2
56	27.6	28.9	32.5	34.3	37.2	39.8	42.9	47.0	50.0	52.7	55.3	58.0	61.0	64.7	69.9	74.5	78.6	83.5	87.0	94.5	97.5
57	28.2	29.6	33.2	35.1	38.0	40.6	43.8	47.9	51.0	53.7	56.3	59.1	62.1	65.7	71.0	75.6	79.8	84.7	88.2	95.8	98.8
58	28.9	30.3	34.0	35.9	38.8	41.5	44.7	48.8	51.9	54.7	57.3	60.1	63.1	66.8	72.2	76.8	80.9	86.0	89.5	97.0	100.1
59	29.6	31.0	34.8	36.7	39.7	42.3	45.6	49.7	52.9	55.6	58.3	61.1	64.2	67.9	73.3	77.9	82.1	87.2	90.7	98.3	101.4
60	30.3	31.7	35.5	37.5	40.5	43.2	46.5	50.6	53.8	56.6	59.3	62.1	65.2	69.0	74.4	79.1	83.3	88.4	92.0	99.6	102.7
61	31.0	32.5	36.3	38.3	41.3	44.0	47.3	51.6	54.8	57.6	60.3	63.2	66.3	70.0	75.5	80.2	84.5	89.6	93.2	100.9	104.0
62	31.7	33.2	37.1	39.1	42.1	44.9	48.2	52.5	55.7	58.6	61.3	64.2	67.3	71.1	76.6	81.4	85.7	90.8	94.4	102.2	105.3
63	32.5	33.9	37.8	39.9	43.0	45.7	49.1	53.4	56.7	59.6	62.3	65.2	68.4	72.2	77.7	82.5	86.8	92.0	95.6	103.4	106.6
64	33.2	34.6	38.6	40.6	43.8	46.6	50.0	54.3	57.6	60.5	63.3	66.2	69.4	73.3	78.9	83.7	88.0	93.2	96.9	104.7	107.9
65	33.9	35.4	39.4	41.4	44.6	47.4	50.9	55.3	58.6	61.5	64.3	67.2	70.5	74.4	80.0	84.8	89.2	94.4	[illegible]	106.0	109.2
66	34.6	36.1	40.2	42.2	45.4	48.3	51.8	56.2	59.5	62.5	65.3	68.3	71.5	75.4	81.1	86.0	90.3	95.6	99.3	107.3	110.5
67	35.3	36.8	40.9	43.0	46.3	49.2	52.7	57.1	60.5	63.5	66.3	69.3	72.6	76.5	82.2	87.1	91.5	96.8	100.6	108.5	111.7
68	36.0	37.6	41.7	43.8	47.1	50.0	53.5	58.0	61.4	64.4	67.3	70.3	73.6	77.6	83.3	88.3	92.7	98.0	101.8	109.8	113.0
69	36.7	38.3	42.5	44.6	47.9	50.9	54.4	59.0	62.4	65.4	68.3	71.3	74.6	78.6	84.4	89.4	93.9	99.2	103.0	111.1	114.3
70	37.5	39.0	43.3	45.4	48.8	51.7	55.3	59.9	63.3	66.4	69.3	72.4	75.7	79.7	85.5	90.5	95.0	100.4	104.2	112.3	115.6
71	38.2	39.8	44.1	46.2	49.6	52.6	56.2	60.8	64.3	67.4	70.3	73.4	76.7	80.8	86.6	91.7	96.2	101.6	105.4	113.6	116.9
72	38.9	40.5	44.8	47.1	50.4	53.5	57.1	61.8	65.3	68.4	71.3	74.4	77.8	81.9	87.7	92.8	97.4	102.8	106.6	114.8	118.1
73	39.6	41.3	45.6	47.9	51.3	54.3	58.0	62.7	66.2	69.3	72.3	75.4	78.8	82.9	88.8	93.9	98.5	104.0	107.9	116.1	119.4
74	40.4	42.0	46.4	48.7	52.1	55.2	58.9	63.6	67.2	70.3	73.3	76.4	79.9	84.0	90.0	95.1	99.7	105.2	109.1	117.3	120.7
75	41.1	42.8	47.2	49.5	52.9	56.1	59.8	64.5	68.1	71.3	74.3	77.5	80.9	85.1	91.1	96.2	100.8	106.4	110.3	118.6	121.9
76	41.8	43.5	48.0	50.3	53.8	56.9	60.7	65.5	69.1	72.3	75.3	78.5	82.0	86.1	92.2	97.4	102.0	107.6	111.5	119.9	123.2
77	42.6	44.3	48.8	51.1	54.6	57.8	61.6	66.4	70.0	73.2	76.3	79.5	83.0	87.2	93.3	98.5	103.2	108.8	112.7	121.1	124.5
78	43.3	45.0	49.6	51.9	55.5	58.7	62.5	67.3	71.0	74.2	77.3	80.5	84.0	88.3	94.4	99.6	104.3	110.0	113.9	122.3	125.7
79	44.1	45.8	50.4	52.7	56.3	59.5	63.4	68.3	72.0	75.2	78.3	81.5	85.1	89.3	95.5	100.7	105.5	111.1	115.1	123.6	127.0
80	44.8	46.5	51.2	53.5	57.2	60.4	64.3	69.2	72.9	76.2	79.3	82.6	86.1	90.4	96.6	101.9	106.6	112.3	116.3	124.8	128.3
81	45.5	47.3	52.0	54.4	58.0	61.3	65.2	70.1	73.9	77.2	80.3	83.6	87.2	91.5	97.7	103.0	107.8	113.5	117.5	126.1	129.5
82	46.3	48.0	52.8	55.2	58.8	62.1	66.1	71.1	74.8	78.1	81.3	84.6	88.2	92.5	98.8	104.1	108.9	114.7	118.7	127.3	130.8
83	47.0	48.8	53.6	56.0	59.7	63.0	67.0	72.0	75.8	79.1	82.3	85.6	89.2	93.6	99.9	105.3	110.1	115.9	119.9	128.6	132.0
84	47.8	49.6	54.4	56.8	60.5	63.9	67.9	72.9	76.8	80.1	83.3	86.6	90.3	94.7	101.0	106.4	111.2	117.1	121.1	129.8	133.3
85	48.5	50.3	55.2	57.6	61.4	64.7	68.8	73.9	77.7	81.1	84.3	87.7	91.3	95.7	102.1	107.5	112.4	118.2	122.3	131.0	134.5
86	49.3	51.1	56.0	58.5	62.2	65.6	69.7	74.8	78.7	82.1	85.3	88.7	92.4	96.8	103.2	108.6	113.5	119.4	123.5	132.3	135.8
87	50.0	51.9	56.8	59.3	63.1	66.5	70.6	75.7	79.6	83.0	86.3	89.7	93.4	97.9	104.3	109.8	114.7	120.6	124.7	133.5	137.0
88	50.8	52.6	57.6	60.1	63.9	67.4	71.5	76.7	80.6	84.0	87.3	90.7	94.4	98.9	105.4	110.9	115.8	121.8	125.9	134.7	138.3
89	51.5	53.4	58.4	60.9	64.8	68.2	72.4	77.6	81.6	85.0	88.3	91.7	95.5	100.0	106.5	112.0	117.0	122.9	127.1	136.0	139.5
90	52.3	54.2	59.2	61.8	65.6	69.1	73.3	78.6	82.5	86.0	89.3	92.8	96.5	101.1	107.6	113.1	118.1	124.1	128.3	137.2	140.8
91	53.0	54.9	60.0	62.6	66.5	70.0	74.2	79.5	83.5	87.0	90.3	93.8	97.6	102.1	108.7	114.3	119.3	125.3	129.5	138.4	142.0
92	53.8	55.7	60.8	63.4	67.4	70.9	75.1	80.4	84.4	88.0	91.3	94.8	98.6	103.2	109.8	115.4	120.4	126.5	130.7	139.7	143.3
93	54.5	56.5	61.6	64.2	68.2	71.8	76.0	81.4	85.4	88.9	92.3	95.8	99.6	104.2	110.9	116.5	121.6	127.6	131.9	140.9	144.5
94	55.3	57.2	62.4	65.1	69.1	72.6	76.9	82.3	86.4	89.9	93.3	96.8	100.7	105.3	111.9	117.6	122.7	128.8	133.1	142.1	145.8
95	56.1	58.0	63.2	65.9	69.9	73.5	77.8	83.2	87.3	90.9	94.3	97.9	101.7	106.4	113.0	118.8	123.9	130.0	134.2	143.3	147.0
96	56.8	58.8	64.1	66.7	70.8	74.4	78.7	84.2	88.3	91.9	95.3	98.9	102.8	107.4	114.1	119.9	125.0	131.1	135.4	144.6	148.2
97	57.6	59.6	64.9	67.6	71.6	75.3	79.6	85.1	89.2	92.9	96.3	99.9	103.8	108.5	115.2	121.0	126.1	132.3	136.6	145.8	149.5
98	58.4	60.4	65.7	68.4	72.5	76.2	80.5	86.1	90.2	93.8	97.3	100.9	104.8	109.5	116.3	122.1	127.3	133.5	137.8	147.0	150.7
99	59.1	61.1	66.5	69.2	73.4	77.0	81.4	87.0	91.2	94.8	98.3	101.9	105.9	110.6	117.4	123.2	128.4	134.6	139.0	148.2	151.9
100	59.9	61.9	67.3	70.1	74.2	77.9	82.4	87.9	92.1	95.8	99.3	102.9	106.9	111.7	118.5	124.3	129.6	135.8	140.2	149.4	153.2

Appendix 2
THE WEIBULL DISTRIBUTION

The Weibull function is an extremely versatile model for describing the proportion of a population that will fail with time. The general form of the Weibull failure density function is:

$$f(x) = a \cdot b \cdot x^{(b-1)} \cdot e^{-a \cdot x^{b}}$$

The above function expresses the probability of failure in the next small increment of time. For reliability applications, the Weibull failure density function is integrated to model the cumulative failures over time. In order to present the equations consistent with this handbook, the above variables are defined as:

$$a = \left(\frac{1}{\theta}\right)^{m}$$

$$b = m$$

$$x = t$$

this gives

$$f(t) = \left(\frac{1}{\theta}\right)^{m}(m)\left(t^{m-1}\right)e^{-\left(\frac{1}{\theta}\right)^{m} \cdot t^{m}}$$

Integrating this expression gives

$$F(t) = \int f(t)\,dt$$

$$= c - e^{-\left(\frac{1}{\theta}\right)^{m} \cdot t^{m}} = c - e^{-\left(\frac{t}{\theta}\right)^{m}}$$

For reliability work c is defined as unity and the expression for the probability of failure is:

$$F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{m}}$$

The parameter, m, is called the shape parameter and the parameter, θ, is called the scale parameter or (less frequently) the characteristic value. When there is a substantial time period before any degradation failures occur, it may be necessary to add a location parameter, γ . In this case the Weibull failure distribution is expressed as:

$$F(t) = 1 - e^{-\left(\frac{t-\gamma}{\theta-\gamma}\right)^m}$$

The location parameter, γ , has been established as zero for the computations and discussions contained in this report. γ shifts the location of the distribution in time but does not change its general shape.

A special case of the Weibull distribution occurs when the shape parameter equals one (m=1). Then the Weibull becomes the exponential distribution and the scale parameter, θ , is commonly called the mean time to failure (MTTF) or the mean time between failures (MTBF). While θ is truly a mean time between failures for the exponential distribution, this is not true for the Weibull distribution in general. Care should be taken to not ascribe more meaning to θ than actually exists.

The Weibull distribution is extremely flexible and versatile in its shape. At m=3.2 the shape is very close to a normal distribution. Figure 25 illustrates some typical shapes.

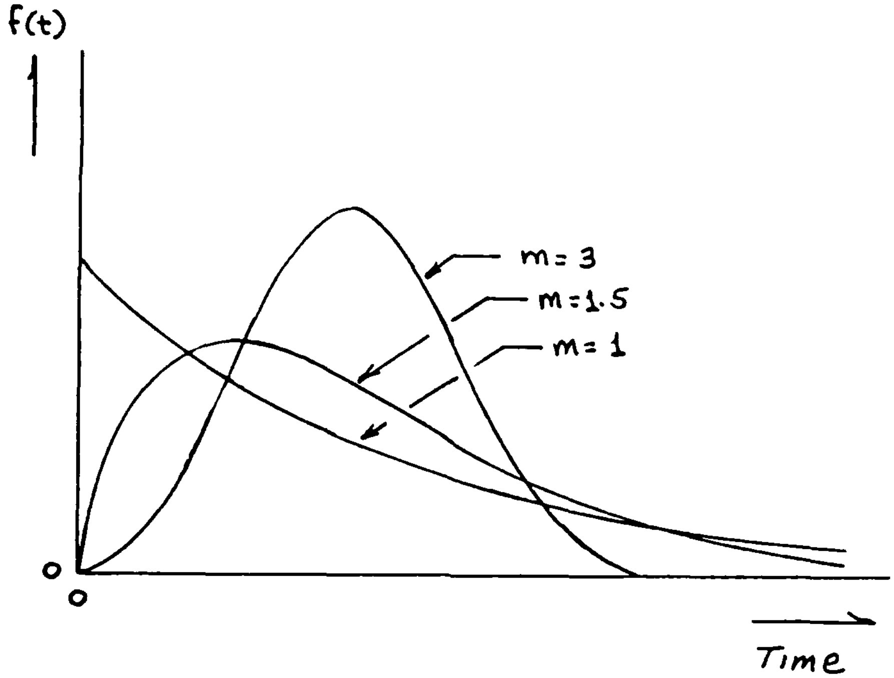


Figure 25

How Shape Varies with the Shape Parameter

If a sample of components is drawn from a population where the time to failure is exactly described by a Weibull distribution, the time versus reliability may be plotted as a straight line on special graph paper similar to figure 26. Lacking this paper, standard rectangular graph paper may be used where the coordinates are:

$$\ln(t) \text{ and } \ln\ln\left[1/R(t)\right]$$

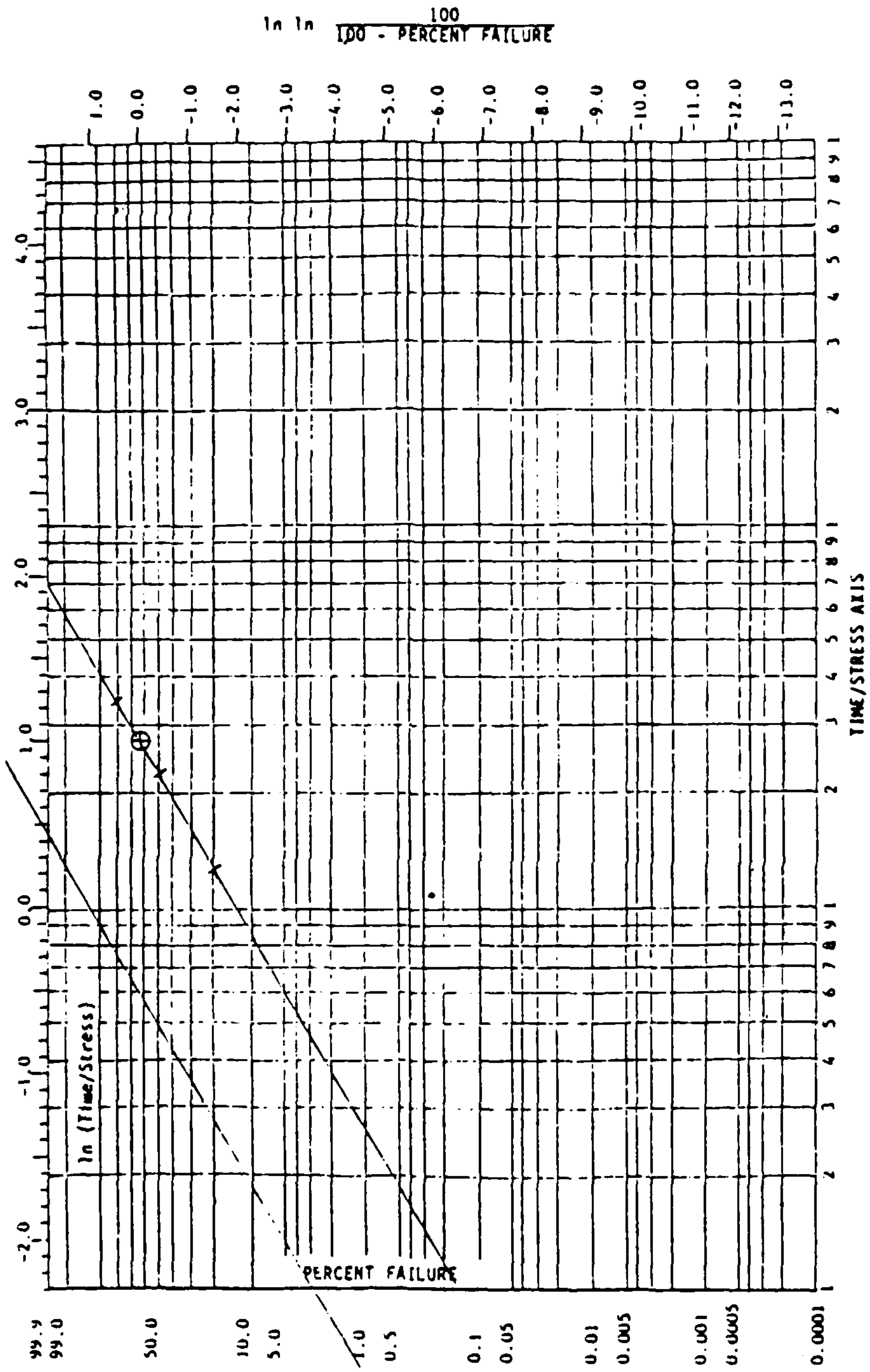


Figure 26

An Example of Weibull Graph Paper

Source: Reference 8

Of course, the engineer never knows the true reliability, R(t), to use in plotting figure 26. For small samples, tested without replacement, the best estimate of R(t) is:

$$E\left[R(t)\right] = 1 - \frac{r}{n+1}$$

where

r	=	number failing
n	=	sample size
t	=	time of the rth failure

As the sample size decreases, there can be considerable error in obtaining a set of points conforming to the true linear relationship. Still, the engineer can obtain a reasonable subjective opinion as to the nature of the population from the plot.

Typically, the Weibull distribution is defined by its reliability or survival function .

$$\begin{aligned} R(t) &= 1 - F(t) \\ &= e^{-\left(\frac{t}{\theta}\right)^m} \end{aligned}$$

The probability of failure in the next small increment of time divided by the probability of surviving up to that time is known as the hazard rate, $z(t)$. The hazard rate is thus the failure density function divided by the reliability function. In the case of Weibull

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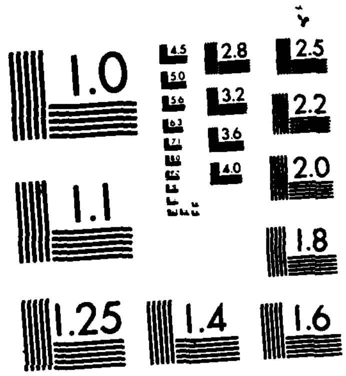

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

$$
\begin{aligned}
Z(t) &= \frac{f(t)}{R(t)} \\
&= \frac{\left(\frac{1}{\theta}\right)^{m}(m)\left(t^{m-1}\right)e^{-\left(\frac{t}{\theta}\right)^{m}}}{e^{-\left(\frac{t}{\theta}\right)^{m}}} \\
&= m\left(\frac{1}{\theta}\right)\left(\frac{1}{\theta}\right)^{m-1}\left(t^{m-1}\right) \\
&= \frac{m}{\theta}\left(\frac{t}{\theta}\right)^{m-1}
\end{aligned}
$$

Figure 27 illustrates how the hazard rate varies with different shape parameters.

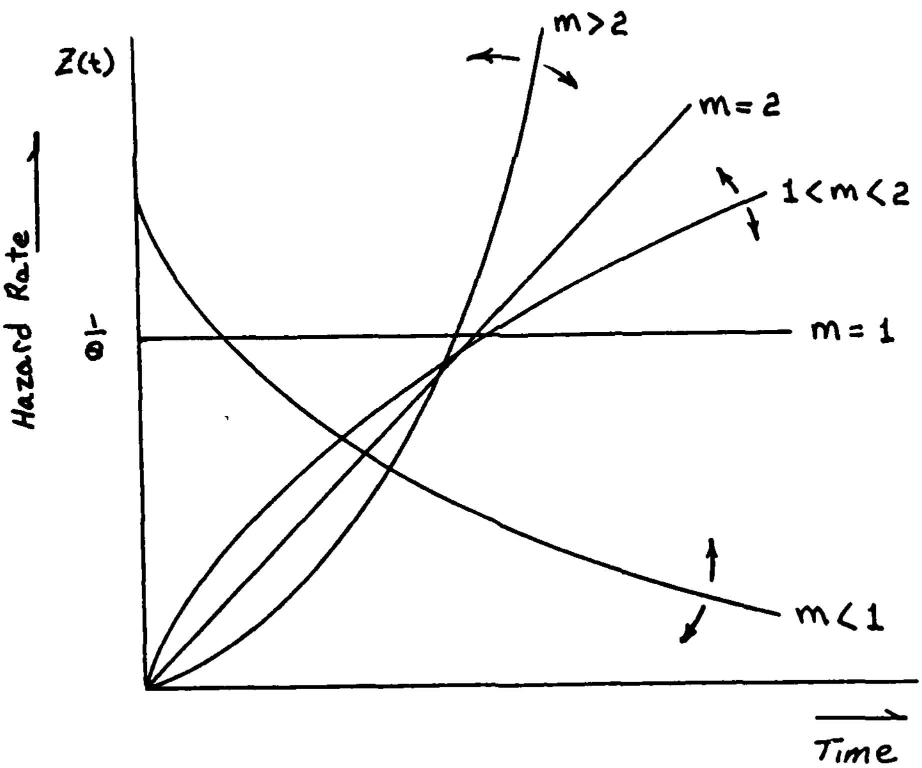

Figure 27
Weibull Hazard Rate Vs Shape Parameter

This report uses three derivations as follows: 1. A derivation of the ratio of test termination times for a Weibull distribution and an exponential distribution. Both have the same reliability requirement and both terminate with the same probability of failure. 2. A derivation for determining the shape parameter from any two sets of time and reliability associated with any two points on the Weibull cumulative failure distribution. This derivation is the basis for the coordinates of the Weibull reliability graph paper discussed earlier. 3. A derivation of the ratio of the test time, at which no failures are to have occurred, to the time at which a reliability requirement is specified, assuming a given sample size and Weibull shape parameter. This derivation is the basis for Appendix 6.

1. Derivation of relative test times for Weibull distributions compared to an exponential distribution.

R_t = Desired Reliability at time t

R_T = Reliability of an exponential distribution at a predetermined test termination time, T

R_{T_m} = Reliability of a Weibull distribution with shape parameter, m, at a test termination time, T_m, such that $R_{T_m} = R_T$

To determine $\frac{T_m}{T}$

Let Θ = MTBF, exponential

Let Θ_m = Scale Parameter for Weibull

at time, t:

$$1 - e^{-t/\theta} = 1 - e^{-(t/\theta_m)^m} = 1 - R_t$$

$$e^{-t/\theta} = e^{-(t/\theta_m)^m} = R_t$$

$$-\frac{t}{\theta} = -\left(\frac{t}{\theta_m}\right)^m = \ln R_t$$

$$\frac{t}{\theta} = -\ln R_t$$

$$\frac{t}{\theta_m} = \left[-\ln R_t\right]^{1/m}$$

$$\frac{\theta_m}{\theta} = \frac{-\ln R_t}{\left[-\ln R_t\right]^{1/m}} = \left[-\ln R_t\right]^{1-1/m}$$

At Test Termination:

$$R_T = R_{T_m}$$

therefore

$$\frac{T}{\theta} = \left[\frac{T_m}{\theta_m}\right]^m = \left[-\ln R_T\right] \qquad \text{As determined earlier}$$

and $$\frac{T_m}{\theta_m} = \left[-\ln R_T\right]^{1/m}$$

$$\frac{\frac{T_m}{\theta_m}}{\frac{T}{\theta}} = \frac{\left[-\ln R_T\right]^{1/m}}{-\ln R_T} \qquad \text{Substituting } \frac{\theta_m}{\theta} = (-\ln R_t)^{1-1/m}$$

$$= \left[(-\ln R_t)^{1-1/m}\right]\left[(-\ln R_T)^{1/m-1}\right]$$

then

$$\frac{T_m}{T} = \frac{(-\ln R_t)^{1-1/m}}{(-\ln R_T)^{1-1/m}}$$

$$\frac{T_m}{T} = \left[\frac{\ln R_t}{\ln R_T}\right]^{1-1/m}$$

2. Derivation of formula for the shape parameter of a Weibull distribution with unknown scale parameter:

$$R(t) = e^{-(t/\theta)^m} = \frac{1}{e^{(t/\theta)^m}} \qquad \text{By definition}$$

$$e^{(t/\theta)^m} = \frac{1}{R(t)}$$

$$(t/\theta)^m = \ln\left[1/R(t)\right]$$

$$m \cdot \ln\left(\frac{t}{\theta}\right) = \ln\ln\left[1/R(t)\right]$$

$$m\left[\ln(t) - \ln(\theta)\right] = \ln\ln\left[1/R(t)\right]$$

Subtracting two failures times, t_2 and t, where $t_2 > t_1$ gives:

$$m\left[\ln(t_2) - \ln(\theta)\right] - m\left[\ln(t_1) - \ln(\theta)\right] = \ln\ln\left[1/R(t_2)\right] - \ln\ln\left[1/R(t_1)\right]$$

$$m\left[\ln(t_2) - \ln(t_1)\right] = \ln\ln\left[1/R(t_2)\right] - \ln\ln\left[1/R(t_1)\right]$$

$$m = \frac{\ln\ln\left[\frac{1}{R(t_2)}\right] - \ln\ln\left[\frac{1}{R(t_1)}\right]}{\ln(t_2) - \ln(t_1)}$$

For estimating R(t), let:

$$R(t) = 1 - \frac{r}{n+1}$$

where r = number of failures at time, t

n = sample size

3. Derivation of the ratio of the test time, at which no failures are to have occurred, to the time at which a reliability requirement is specified, assuming a given sample size and Weibull shape parameter:

R_t = Minimum acceptable reliability at time, t.

R_T = Reliability of a Weibull distribution with no failures at time T.

θ = Lowest acceptable Weibull scale parameter

m = Weibull shape parameter

β = Consumers risk - probability of accepting a lot whose scale parameter is equal to or less than θ .

n = Sample size

Given $$R_t = e^{-(t/\theta)^m}$$

$$-\ln R_t = (t/\theta)^m$$

then $$t = \theta(-\ln R_t)^{1/m}$$

Given $$R_T = e^{-(T/\theta)^m} = \beta^{1/n}$$

$$-\ln(\beta^{1/n}) = (T/\theta)^m$$

$$\frac{-\ln \beta}{n} = (T/\theta)^m \quad \text{since} \quad \ln(\beta^{1/n}) = \frac{\ln \beta}{n}$$

then $$T = \theta\left[\frac{-\ln \beta}{n}\right]^{1/m}$$

and $$\frac{T}{t} = \frac{\theta\left[\frac{-\ln \beta}{n}\right]^{1/m}}{\theta\left[-\ln R_t\right]^{1/m}} = \left[\frac{\ln \beta}{n \cdot \ln R_t}\right]^{1/m}$$

APPENDIX 3

PROBABILITY RATIO SEQUENTIAL TEST FORMULAS

The formulas of Appendix 3 may be used to compute the accept and reject boundaries for a Probability Ratio Sequential Test on a variety of failure distribution models. α = risk of accepting A when H is true. β = risk of accepting H when A is true. Note that within the convention of MIL-STD-781C, and the convention used in this report, the lower subscript is larger. Thus, $\theta_1 < \theta_0$ in USAF practice, whereas $\theta_1 > \theta_0$ in Appendix 3. This results in the conventional usage of consumers risk, the probability of erroneously accepting the lower MTBF, being consistent with α as used in this Appendix. The Appendix 3 usage of β is consistent with the producer risk as used elsewhere in this handbook.

Sources: References 5 and 8.

Hypothesis and Alternative	Assumptions	Test Statistic	Acceptance Value	Rejection Value
H: $p = P_0$ A: $p = P_1$	Binomial $P_1 > P_0$	X = observations associated with P	$\dfrac{\ln[\beta/(1-\alpha)] + n\cdot\ln[(1-P_0)/(1-P_1)]}{\ln(P_1/P_0) - \ln[(1-P_1)/(1-P_0)]}$	$\dfrac{\ln[(1-\beta)/\alpha] + n\cdot\ln[(1-P_0)/(1-P_1)]}{\ln(P_1/P_0) - \ln[(1-P_1)/(1-P_0)]}$
H: $\sigma = \sigma_0$ A: $\sigma = \sigma_1$	Normal $\sigma_1 > \sigma_0$ μ Known	$\sum (x-\mu)^2$	$\dfrac{2\cdot\ln[\beta/(1-\alpha)]\, n\cdot\ln(\sigma_1^2/\sigma_0^2)}{1/\sigma_0^2 - 1/\sigma_1^2}$	$\dfrac{2\cdot\ln[(1-\beta)/\alpha] + n\cdot\ln(\sigma_1^2/\sigma_0^2)}{1/\sigma_0^2 - 1/\sigma_1^2}$
H: $\sigma = \sigma_0$ A: $\sigma = \sigma_1$	Normal $\sigma_1 > \sigma_0$ μ unknown	$\sum (x-\bar{x})^2$	as above	as above
H: $\mu = \mu_0$ A: $\mu = \mu_1$	Normal σ Known	$\sum x$	$[\sigma^2/(\mu_1-\mu_0)]\ln[\beta/(1-\alpha)] + \dfrac{n(\mu_0+\mu_1)}{2}$	$[\sigma^2/(\mu_1-\mu_0)]\ln[(1-\beta)/\alpha] + \dfrac{n(\mu_0+\mu_1)}{2}$
H: $\lambda = \lambda_0$ A: $\lambda = \lambda_1$	Poisson $\lambda_1 > \lambda_0$	$\sum x$	$\dfrac{\ln[\beta/(1-\alpha)] + n(\lambda_1-\lambda_0)}{\ln\lambda_1 - \ln\lambda_0}$	$\dfrac{\ln[(1-\beta)/\alpha] + n(\lambda_1-\lambda_0)}{\ln\lambda_1 - \ln\lambda_0}$
H: $\theta = \theta_0$ A: $\theta = \theta_1$	Exponential $\theta_1 > \theta_0$	$\sum x$	$\dfrac{-[\ln \beta/(1-\alpha)] + n\cdot\ln(\theta_0/\theta_1)}{1/\theta_1 - 1/\theta_0}$	$\dfrac{-\ln[(1-\beta)/\alpha] + n\cdot\ln(\theta_0/\theta_1)}{1/\theta_1 - 1/\theta_0}$
H: $\theta = \theta_0$ A: $\theta = \theta_1$	Weibull $\theta_1 > \theta_0$ m known	$\sum x^m$	$\dfrac{-\ln[\beta/(1-\alpha)] + n\cdot m\cdot\ln(\theta_0/\theta_1)}{1/\theta_1^m - 1/\theta_0^m}$	$\dfrac{-\ln[(1-\beta)/\alpha] + n\cdot m\cdot\ln(\theta_0/\theta_1)}{1/\theta_1^m - 1/\theta_0^m}$

APPENDIX 4

SIMULATION STUDIES FOR TRUNCATED RELIABILITY DEMONSTRATION TESTS

This Appendix presents the results of a simulation study where random numbers between 0 and 1 were generated on a Radio Shack TRS-80 Model III. To prevent the program from stopping on a divide by zero error code any number less than .00001 was rounded up to .00001 and any number above .99999 was rounded down to .99999. For each sample size, an equivalent number of random numbers were generated and transformed into failure times consistent with a given Weibull shape parameter (enumerated in line three of each table heading). Failure times were then ranked and a simulated shape parameter was calculated per Appendix 2 for each simulation based upon the difference between the time of the first failure (lowest time of failure) and the time of the Rth failure. This was done for up to a total of ten rejection numbers, R, for sample sizes which varied from ten to forty. Each simulation was performed one thousand times and the shape parameters thus estimated which fell below a given cut-off value were summed and divided by the total number of simulations, 1000. This is the three digit number presented in columns below each shape parameter cut-off value, M. This proportion becomes an estimate of the one sided confidence level that, having conducted a test and calculated a shape parameter equal to M for a given sample size and rejection number, the true population shape parameter is at least as large as the value which is enumerated in line three of each table heading as the population shape parameter. Note that the above simulation cannot provide a confidence estimate good to the third decimal place. Additional simulations indicate that this technique is accurate within a range of .006. Appendix 4 is to be used when analyzing data from a truncated test.

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 10
POPULATION SHAPE PARAMETER 1

R	EST. M = 1.5	2.0	2.5	3.0	3.5	4.0
2	.622	.703	.74	.771	.805	.832
3	.717	.823	.873	.902	.926	.941
4	.783	.886	.926	.952	.97	.978
5	.827	.921	.962	.978	.988	.992
6	.854	.948	.981	.994	.997	.999
7	.88	.962	.99	.998	1	1
8	.902	.973	.993	1	1	1
9	.935	.993	.999	1	1	1
10	.951	.993	.999	1	1	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 10
POPULATION SHAPE PARAMETER 1.1

R	EST. M = 1.5	2.0	2.5	3.0	3.5	4.0
2	.611	.69	.761	.799	.832	.858
3	.704	.812	.878	.921	.939	.951
4	.755	.874	.938	.958	.971	.985
5	.783	.921	.961	.98	.986	.991
6	.812	.942	.977	.99	.996	.998
7	.837	.954	.989	.997	.999	1
8	.87	.968	.995	.999	1	1
9	.885	.975	.997	.999	1	1
10	.898	.986	.999	1	1	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 10
POPULATION SHAPE PARAMETER 1.2

R	EST. M = 1.5	2.0	2.5	3.0	3.5	4.0
2	.553	.655	.712	.74	.764	.794
3	.635	.761	.831	.873	.898	.921
4	.686	.826	.894	.926	.949	.964
5	.736	.864	.933	.962	.977	.986
6	.76	.892	.954	.981	.992	.997
7	.778	.924	.973	.99	.998	1
8	.807	.939	.984	.993	.999	1
9	.813	.961	.996	.999	1	1
10	.851	.979	.995	.999	1	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 10
POPULATION SHAPE PARAMETER 1.3

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.545	.635	.686	.727	.757	.777
3	.625	.748	.805	.866	.894	.911
4	.653	.796	.873	.934	.949	.965
5	.68	.846	.923	.961	.975	.984
6	.711	.866	.937	.97	.984	.99
7	.72	.891	.958	.982	.993	.995
8	.746	.91	.967	.987	.994	.998
9	.775	.93	.975	.991	.995	.999
10	.796	.951	.986	.998	.999	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 10
POPULATION SHAPE PARAMETER 1.4

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.535	.618	.68	.723	.759	.786
3	.595	.705	.776	.829	.862	.891
4	.632	.775	.845	.899	.927	.948
5	.647	.82	.886	.936	.96	.972
6	.666	.846	.917	.961	.975	.981
7	.69	.876	.941	.971	.984	.992
8	.718	.893	.957	.976	.99	.995
9	.725	.908	.963	.989	.994	.997
10	.747	.936	.978	.994	.998	.999

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 10
POPULATION SHAPE PARAMETER 1.5

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.504	.6	.669	.719	.772	.795
3	.566	.698	.791	.846	.884	.919
4	.585	.738	.845	.907	.942	.955
5	.585	.767	.886	.941	.963	.978
6	.599	.802	.914	.961	.981	.989
7	.613	.828	.93	.973	.99	.997
8	.615	.861	.951	.982	.997	.999
9	.636	.868	.961	.989	.998	.999
10	.643	.887	.972	.993	1	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 20
POPULATION SHAPE PARAMETER 1

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.629	.705	.749	.786	.808	.823
3	.721	.81	.868	.908	.92	.937
4	.759	.856	.922	.946	.965	.977
5	.797	.899	.947	.97	.988	.992
6	.823	.925	.964	.985	.995	.996
7	.856	.948	.98	.995	.996	.999
8	.869	.951	.986	.995	.999	1
9	.879	.964	.987	.995	1	1
10	.897	.971	.989	.997	1	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 20
POPULATION SHAPE PARAMETER 1.1

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.601	.68	.729	.766	.793	.814
3	.687	.777	.85	.892	.911	.926
4	.722	.835	.899	.936	.954	.967
5	.759	.865	.931	.96	.978	.989
6	.791	.896	.951	.976	.991	.995
7	.809	.927	.967	.986	.995	.996
8	.824	.93	.973	.99	.997	.999
9	.839	.943	.98	.992	.997	1
10	.844	.95	.985	.994	.999	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 20
POPULATION SHAPE PARAMETER 1.2

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.564	.652	.711	.763	.796	.821
3	.671	.779	.839	.879	.91	.933
4	.702	.826	.903	.934	.959	.969
5	.722	.861	.934	.963	.977	.985
6	.743	.889	.945	.975	.985	.992
7	.771	.898	.958	.986	.994	.998
8	.778	.913	.97	.99	.997	.998
9	.781	.926	.98	.996	.997	.998
10	.793	.941	.986	.998	.999	.999

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 20
POPULATION SHAPE PARAMETER 1.3

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.547	.654	.708	.754	.782	.805
3	.637	.767	.83	.88	.903	.922
4	.679	.802	.879	.92	.946	.964
5	.695	.825	.916	.945	.966	.977
6	.717	.855	.934	.966	.982	.991
7	.729	.879	.945	.974	.986	.996
8	.741	.9	.963	.983	.994	.998
9	.756	.919	.972	.988	.993	.998
10	.76	.931	.976	.993	.997	.998

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 20
POPULATION SHAPE PARAMETER 1.4

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.534	.612	.675	.716	.749	.774
3	.595	.699	.768	.832	.868	.905
4	.612	.741	.831	.88	.922	.937
5	.625	.777	.858	.915	.947	.964
6	.646	.808	.892	.938	.964	.977
7	.658	.833	.924	.957	.98	.988
8	.667	.844	.923	.964	.986	.993
9	.681	.857	.936	.973	.987	.995
10	.681	.868	.947	.979	.989	.995

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 20
POPULATION SHAPE PARAMETER 1.5

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.496	.588	.652	.699	.742	.77
3	.556	.692	.779	.827	.864	.895
4	.577	.733	.826	.889	.922	.945
5	.585	.76	.861	.921	.958	.971
6	.588	.781	.889	.936	.968	.981
7	.606	.804	.898	.951	.979	.989
8	.615	.822	.913	.961	.985	.994
9	.609	.823	.926	.972	.993	.997
10	.609	.836	.941	.979	.996	.998

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 30
POPULATION SHAPE PARAMETER 1

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.629	.715	.758	.791	.814	.828
3	.739	.833	.888	.911	.932	.949
4	.78	.881	.923	.948	.972	.978
5	.817	.915	.949	.975	.985	.991
6	.85	.933	.966	.985	.991	.995
7	.855	.948	.978	.99	.995	.998
8	.872	.96	.984	.996	.998	.999
9	.889	.967	.988	.995	.998	.999
10	.902	.976	.991	.998	.999	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 30
POPULATION SHAPE PARAMETER 1.1

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.604	.681	.74	.773	.803	.826
3	.708	.798	.871	.903	.92	.94
4	.736	.853	.914	.943	.958	.969
5	.773	.892	.944	.968	.981	.989
6	.789	.916	.965	.981	.991	.994
7	.808	.932	.973	.987	.994	.994
8	.829	.944	.978	.99	.995	.998
9	.854	.956	.982	.993	.997	.998
10	.858	.964	.986	.996	.998	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 30
POPULATION SHAPE PARAMETER 1.2

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.585	.659	.722	.758	.785	.808
3	.668	.785	.854	.888	.909	.926
4	.717	.823	.895	.923	.947	.964
5	.73	.857	.923	.949	.974	.98
6	.747	.882	.938	.966	.982	.99
7	.765	.9	.955	.978	.989	.993
8	.78	.914	.967	.984	.996	.997
9	.805	.925	.975	.988	.994	.998
10	.806	.934	.98	.991	.996	.999

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 30
POPULATION SHAPE PARAMETER 1.3

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.548	.631	.695	.744	.769	.797
3	.62	.753	.815	.873	.901	.915
4	.659	.789	.869	.916	.941	.957
5	.668	.832	.905	.946	.966	.977
6	.684	.856	.928	.969	.981	.991
7	.698	.87	.948	.973	.987	.994
8	.717	.885	.954	.979	.99	.993
9	.731	.897	.964	.984	.993	.996
10	.743	.914	.969	.987	.995	.998

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 30
POPULATION SHAPE PARAMETER 1.4

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.524	.616	.676	.724	.76	.782
3	.587	.721	.794	.857	.886	.907
4	.614	.76	.846	.897	.925	.951
5	.621	.795	.89	.933	.958	.974
6	.636	.815	.911	.954	.978	.985
7	.65	.837	.925	.97	.981	.99
8	.664	.852	.942	.975	.986	.99
9	.673	.868	.948	.978	.99	.995
10	.672	.884	.958	.983	.992	.996

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 30
POPULATION SHAPE PARAMETER 1.5

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.519	.601	.659	.715	.746	.769
3	.575	.697	.785	.833	.874	.898
4	.592	.739	.823	.881	.913	.936
5	.589	.762	.857	.915	.941	.961
6	.604	.784	.882	.933	.954	.975
7	.613	.81	.9	.948	.969	.982
8	.616	.821	.914	.96	.981	.992
9	.624	.84	.925	.967	.984	.992
10	.629	.845	.934	.976	.99	.994

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 40
POPULATION SHAPE PARAMETER 1

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.627	.715	.76	.804	.822	.838
3	.73	.821	.87	.904	.92	.933
4	.773	.872	.918	.941	.959	.966
5	.802	.904	.938	.961	.971	.98
6	.826	.927	.96	.976	.987	.99
7	.838	.946	.974	.986	.992	.995
8	.865	.953	.98	.99	.994	.997
9	.876	.965	.987	.993	.996	.999
10	.888	.974	.989	.998	.998	1

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 40
POPULATION SHAPE PARAMETER 1.1

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.605	.691	.742	.784	.811	.826
3	.696	.794	.853	.889	.914	.923
4	.736	.839	.901	.931	.949	.96
5	.766	.878	.932	.95	.966	.973
6	.784	.895	.953	.967	.981	.989
7	.797	.92	.962	.981	.988	.994
8	.816	.933	.971	.986	.99	.994
9	.827	.934	.977	.988	.994	.998
10	.834	.951	.983	.994	.998	.999

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 40
POPULATION SHAPE PARAMETER 1.2

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.573	.656	.723	.76	.8	.818
3	.66	.771	.834	.87	.902	.917
4	.7	.814	.885	.918	.938	.956
5	.725	.849	.916	.938	.959	.97
6	.738	.869	.934	.96	.975	.983
7	.745	.89	.953	.974	.984	.991
8	.758	.902	.96	.98	.989	.992
9	.771	.912	.967	.987	.992	.996
10	.781	.927	.975	.989	.997	.998

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 40
POPULATION SHAPE PARAMETER 1.3

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.554	.632	.705	.743	.778	.806
3	.618	.742	.809	.857	.885	.911
4	.658	.783	.86	.906	.93	.946
5	.677	.82	.897	.933	.948	.963
6	.693	.833	.916	.955	.967	.98
7	.706	.853	.934	.967	.979	.988
8	.712	.872	.945	.973	.986	.99
9	.719	.888	.953	.981	.988	.993
10	.726	.897	.967	.983	.994	.998

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 40
POPULATION SHAPE PARAMETER 1.4

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.525	.617	.683	.728	.76	.796
3	.585	.714	.789	.838	.87	.897
4	.618	.754	.834	.889	.918	.935
5	.634	.783	.875	.919	.938	.955
6	.656	.805	.89	.938	.96	.974
7	.657	.817	.916	.956	.974	.983
8	.661	.846	.928	.964	.98	.989
9	.665	.85	.934	.972	.987	.991
10	.669	.861	.945	.976	.989	.997

TOTAL TRIALS SIMULATED 1000
SAMPLE SIZE 40
POPULATION SHAPE PARAMETER 1.5

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.504	.598	.656	.715	.744	.774
3	.558	.686	.771	.821	.858	.884
4	.578	.727	.814	.872	.906	.928
5	.595	.76	.849	.904	.933	.947
6	.6	.771	.869	.927	.956	.966
7	.605	.786	.89	.946	.967	.974
8	.618	.805	.902	.953	.974	.986
9	.614	.814	.912	.965	.981	.988
10	.617	.823	.927	.974	.984	.993

APPENDIX 5

SIMULATION STUDIES FOR PROBABILITY RATIO SEQUENTIAL TESTS

This Appendix presents the results of a simulation study where random numbers between 0 and 1 were generated on a Radio Shack TRS-80 Model III. To prevent the program from stopping on a divide by zero error code any number less than .00001 was rounded up to .00001 and any number above .99999 was rounded down to .99999. For each rejection number, R, an equivalent number of random numbers were generated and transformed into failure times consistent with a given Weibull shape parameter (enumerated in line three of each table heading). Failure times were then ranked and a simulated shape parameter was calculated per Appendix 2 for each simulation based upon the difference between the time of the first failure (lowest time of failure) and the time of the final failure, R. This was done for up to a total of ten rejection numbers. Each simulation was performed one thousand times and the shape parameters thus estimated which fell below a given cut-off value were summed and divided by the total number of simulations, 1000. This is the three digit number presented in columns below each shape parameter cut-off value, M. This proportion becomes an estimate of the one sided confidence level that, having conducted a test and calculated a shape parameter equal to M, the true population shape parameter is at least as large as the value which is enumerated in line three of each table heading as the population shape parameter. Note that the above simulation cannot provide a confidence estimate good to the third decimal place. Additional simulations indicate that this technique is accurate within a range of .006. Simulations in Appendix 5 differ from Appendix 4 in that, for each iteration, the sample size equals the rejection number whereas in Appendix 4 the sample size may be larger than the rejection number. Appendix 5 is to be used only when analyzing data from a Probability Ratio Sequential Test (PRST).

TOTAL TRIALS SIMULATED 1000
MAXIMUM NUMBER OF FAILURES 10
POPULATION SHAPE PARAMETER 1

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.661	.745	.791	.82	.848	.874
3	.782	.857	.916	.939	.954	.965
4	.85	.914	.954	.977	.989	.991
5	.875	.946	.975	.983	.994	.997
6	.911	.976	.988	.995	.997	.998
7	.904	.971	.987	.994	.997	.999
8	.94	.988	.993	.999	.999	1
9	.934	.99	.999	1	1	1
10	.95	.989	.998	1	1	1

TOTAL TRIALS SIMULATED 1000
MAXIMUM NUMBER OF FAILURES 10
POPULATION SHAPE PARAMETER 1.1

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.638	.721	.774	.809	.831	.858
3	.747	.837	.891	.926	.943	.957
4	.814	.9	.941	.963	.984	.99
5	.838	.931	.969	.978	.99	.996
6	.881	.961	.985	.99	.996	.997
7	.869	.957	.982	.99	.994	.997
8	.897	.974	.991	.997	.999	1
9	.892	.977	.995	1	1	1
10	.904	.979	.996	1	1	1

TOTAL TRIALS SIMULATED 1000
MAXIMUM NUMBER OF FAILURES 10
POPULATION SHAPE PARAMETER 1.2

EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
R						
2	.611	.694	.756	.791	.814	.837
3	.704	.812	.866	.916	.936	.95
4	.763	.877	.922	.954	.972	.985
5	.803	.909	.955	.975	.982	.991
6	.836	.938	.98	.988	.993	.996
7	.811	.931	.974	.987	.993	.995
8	.854	.964	.989	.993	.999	.999
9	.84	.958	.993	.999	1	1
10	.854	.971	.994	.998	1	1

READY

TOTAL TRIALS SIMULATED 1000
MAXIMUM NUMBER OF FAILURES 10
POPULATION SHAPE PARAMETER 1.3

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.583	.674	.739	.775	.807	.823
3	.668	.79	.849	.893	.924	.941
4	.711	.858	.907	.944	.963	.979
5	.739	.885	.941	.97	.978	.987
6	.79	.92	.972	.986	.99	.996
7	.762	.914	.968	.983	.989	.994
8	.799	.947	.982	.991	.996	.999
9	.781	.942	.983	.996	1	1
10	.797	.953	.987	.996	.999	1

TOTAL TRIALS SIMULATED 1000
MAXIMUM NUMBER OF FAILURES 10
POPULATION SHAPE PARAMETER 1.4

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.6	.691	.746	.797	.827	.854
3	.648	.775	.839	.876	.906	.924
4	.683	.826	.89	.942	.964	.972
5	.673	.861	.927	.958	.972	.984
6	.68	.859	.948	.974	.989	.995
7	.69	.893	.957	.981	.992	.997
8	.73	.907	.972	.993	.997	.999
9	.732	.924	.978	.994	.999	.999
10	.73	.912	.981	.993	.997	1

TOTAL TRIALS SIMULATED 1000
MAXIMUM NUMBER OF FAILURES 10
POPULATION SHAPE PARAMETER 1.5

R \ EST. M =	1.5	2.0	2.5	3.0	3.5	4.0
2	.536	.629	.694	.745	.776	.806
3	.581	.737	.812	.857	.894	.924
4	.625	.8	.877	.914	.947	.963
5	.641	.829	.909	.946	.97	.978
6	.679	.868	.938	.976	.987	.99
7	.646	.859	.931	.971	.984	.989
8	.68	.887	.964	.988	.992	[illegible]
9	.636	.885	.958	.99	.996	1
10	.658	.895	.971	.989	.996	.999

APPENDIX 6

ZERO FAILURE ACCEPTANCE CRITERIA

Appendix 6 presents the ratio of the test time, during which no failure is to occur to that time at which a given reliability requirement is specified. This ratio is presented for a variety of sample sizes, consumer risks, and Weibull shape parameters. As an example, for a sample size of ten and a consumer risk of 0.05, items from a population with a Weibull shape parameter of 1.5 would have to not fail in 3.25 hours of operation to demonstrate a reliability objective of .95 at the end of one hour of operation.

SAMPLE SIZE = 10 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	299.47	178.34	115.78	80.34	58.73	44.77
.99	29.81	21.9	16.93	13.62	11.31	9.62
.98	14.83	11.61	9.47	7.96	6.87	6.04
.97	9.84	7.99	6.72	5.81	5.12	4.6
.96	7.34	6.13	5.27	4.64	4.16	3.78
.95	5.85	4.98	4.36	3.89	3.53	3.25
.94	4.85	4.2	3.73	3.37	3.09	2.87
.93	4.13	3.63	3.26	2.98	2.76	2.58
.92	3.6	3.2	2.91	2.68	2.5	2.35
.91	3.18	2.86	2.62	2.44	2.29	2.17
.9	2.85	2.59	2.39	2.24	2.11	2.01

SAMPLE SIZE = 15 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	199.65	123.36	82.59	58.81	43.96	34.16
.99	19.88	15.15	12.08	9.97	8.46	7.34
.98	9.89	8.03	6.75	5.83	5.14	4.61
.97	6.56	5.53	4.8	4.25	3.84	3.51
.96	4.9	4.24	3.76	3.4	3.11	2.89
.95	3.9	3.45	3.11	2.85	2.65	2.48
.94	3.23	2.91	2.66	2.47	2.31	2.19
.93	2.76	2.52	2.33	2.18	2.07	1.97
.92	2.4	2.22	2.08	1.96	1.87	1.8
.91	2.12	1.98	1.87	1.79	1.71	1.65
.9	1.9	1.79	1.71	1.64	1.58	1.54

SAMPLE SIZE = 20 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	149.74	94.97	64.98	47.14	35.8	28.2
.99	14.91	11.66	9.51	7.99	6.89	6.06
.98	7.42	6.18	5.31	4.67	4.19	3.81
.97	4.92	4.26	3.78	3.41	3.12	2.9
.96	3.67	3.27	2.96	2.72	2.54	2.38
.95	2.93	2.65	2.45	2.29	2.15	2.05
.94	2.43	2.24	2.09	1.98	1.89	1.81
.93	2.07	1.94	1.83	1.75	1.68	1.63
.92	1.8	1.71	1.63	1.57	1.52	1.48
.91	1.59	1.53	1.48	1.43	1.4	1.37
.9	1.43	1.38	1.35	1.32	1.29	1.27

SAMPLE SIZE = 25 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	119.79	77.53	53.96	39.7	30.52	24.3
.99	11.93	9.52	7.89	6.73	5.88	5.22
.98	5.94	5.05	4.41	3.94	3.57	3.28
.97	3.94	3.48	3.14	2.87	2.67	2.5
.96	2.94	2.67	2.46	2.29	2.16	2.06
.95	2.34	2.17	2.03	1.93	1.84	1.77
.94	1.94	1.83	1.74	1.67	1.61	1.56
.93	1.66	1.58	1.52	1.48	1.44	1.4
.92	1.44	1.4	1.36	1.33	1.3	1.28
.91	1.28	1.25	1.23	1.21	1.19	1.18
.9	1.14	1.13	1.12	1.11	1.1	1.09

SAMPLE SIZE = 30 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	99.83	65.69	46.35	34.51	26.8	21.52
.99	9.94	8.07	6.78	5.85	5.16	4.63
.98	4.95	4.28	3.79	3.42	3.14	2.91
.97	3.28	2.95	2.69	2.5	2.34	2.21
.96	2.45	2.26	2.11	1.99	1.9	1.82
.95	1.95	1.84	1.75	1.67	1.61	1.56
.94	1.62	1.55	1.5	1.45	1.41	1.38
.93	1.38	1.34	1.31	1.28	1.26	1.24
.92	1.2	1.18	1.17	1.15	1.14	1.13
.91	1.06	1.06	1.05	1.05	1.05	1.04
.9	.95	.96	.96	.96	.97	.97

SAMPLE SIZE = 35 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	85.57	57.1	40.76	30.65	24	19.42
.99	8.52	7.01	5.96	5.2	4.62	4.18
.98	4.24	3.72	3.34	3.04	2.81	2.62
.97	2.82	2.56	2.37	2.22	2.1	2
.96	2.1	1.97	1.86	1.77	1.7	1.64
.95	1.67	1.6	1.54	1.49	1.45	1.41
.94	1.39	1.35	1.32	1.29	1.27	1.25
.93	1.18	1.17	1.15	1.14	1.13	1.12
.92	1.03	1.03	1.03	1.03	1.02	1.02
.91	.91	.92	.93	.93	.94	.94
.9	.82	.83	.85	.86	.87	.88

SAMPLE SIZE = 40 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	74.87	50.58	36.47	27.66	21.82	17.77
.99	7.46	6.21	5.34	4.69	4.2	3.82
.98	3.71	3.3	2.98	2.74	2.55	2.4
.97	2.46	2.27	2.12	2	1.91	1.83
.96	1.84	1.74	1.66	1.6	1.55	1.5
.95	1.47	1.42	1.38	1.34	1.32	1.29
.94	1.22	1.19	1.18	1.16	1.15	1.14
.93	1.04	1.03	1.03	1.03	1.03	1.03
.92	.9	.91	.92	.93	.93	.94
.91	.8	.82	.83	.84	.85	.86
.9	.72	.74	.76	.77	.79	.8

SAMPLE SIZE = 45 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	66.55	45.44	33.06	25.26	20.06	16.43
.99	6.63	5.58	4.84	4.29	3.86	3.53
.98	3.3	2.96	2.71	2.51	2.35	2.22
.97	2.19	2.04	1.92	1.83	1.75	1.69
.96	1.64	1.56	1.51	1.46	1.42	1.39
.95	1.3	1.27	1.25	1.23	1.21	1.19
.94	1.08	1.07	1.07	1.06	1.06	1.05
.93	.92	.93	.94	.94	.95	.95
.92	.8	.82	.83	.85	.86	.87
.91	.71	.73	.75	.77	.78	.8
.9	.64	.66	.69	.71	.73	.74

SAMPLE SIZE = 50 CONSUMER RISK = .05

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	59.9	41.29	30.28	23.3	18.61	15.31
.99	5.97	5.07	4.43	3.95	3.58	3.29
.98	2.97	2.69	2.48	2.31	2.18	2.07
.97	1.97	1.85	1.76	1.69	1.63	1.57
.96	1.47	1.42	1.38	1.35	1.32	1.3
.95	1.17	1.16	1.14	1.13	1.12	1.11
.94	.97	.98	.98	.98	.98	.98
.93	.83	.85	.86	.87	.88	.89
.92	.72	.75	.76	.78	.79	.81
.91	.64	.67	.69	.71	.73	.74
.9	.57	.6	.63	.65	.67	.69

SAMPLE SIZE = 10 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	230.18	140.39	92.98	65.62	48.67	37.56
.99	22.92	17.24	13.6	11.13	9.37	8.07
.98	11.4	9.14	7.6	6.51	5.69	5.07
.97	7.56	6.29	5.4	4.74	4.25	3.86
.96	5.65	4.82	4.23	3.79	3.45	3.17
.95	4.49	3.92	3.5	3.18	2.93	2.73
.94	3.73	3.31	2.99	2.75	2.56	2.41
.93	3.18	2.86	2.62	2.44	2.29	2.16
.92	2.77	2.52	2.34	2.19	2.07	1.97
.91	2.45	2.26	2.11	1.99	1.9	1.82
.9	2.19	2.04	1.92	1.83	1.75	1.69

SAMPLE SIZE = 15 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	153.46	97.11	66.32	48.04	36.43	28.67
.99	15.28	11.93	9.7	8.15	7.01	6.16
.98	7.6	6.32	5.42	4.76	4.26	3.87
.97	5.04	4.36	3.85	3.47	3.18	2.94
.96	3.77	3.34	3.02	2.78	2.58	2.42
.95	3	2.71	2.5	2.33	2.19	2.08
.94	2.49	2.29	2.14	2.02	1.92	1.84
.93	2.12	1.98	1.87	1.78	1.71	1.65
.92	1.85	1.75	1.67	1.6	1.55	1.51
.91	1.63	1.56	1.51	1.46	1.42	1.39
.9	1.46	1.41	1.37	1.34	1.31	1.29

SAMPLE SIZE = 20 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	115.09	74.76	52.19	38.5	29.66	23.67
.99	11.46	9.18	7.63	6.53	5.71	5.09
.98	5.7	4.87	4.27	3.82	3.47	3.2
.97	3.78	3.35	3.03	2.79	2.59	2.43
.96	2.83	2.57	2.38	2.23	2.1	2
.95	2.25	2.09	1.97	1.87	1.79	1.72
.94	1.87	1.76	1.68	1.62	1.56	1.52
.93	1.59	1.53	1.47	1.43	1.4	1.37
.92	1.39	1.35	1.31	1.29	1.26	1.24
.91	1.23	1.2	1.19	1.17	1.16	1.15
.9	1.1	1.09	1.08	1.08	1.07	1.07

SAMPLE SIZE = 25 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	92.08	61.04	43.33	32.43	25.29	20.39
.99	9.17	7.5	6.34	5.5	4.87	4.38
.98	4.56	3.98	3.55	3.22	2.96	2.75
.97	3.03	2.74	2.52	2.35	2.21	2.1
.96	2.26	2.1	1.98	1.87	1.79	1.73
.95	1.8	1.71	1.63	1.57	1.52	1.48
.94	1.49	1.44	1.4	1.36	1.33	1.31
.93	1.27	1.25	1.22	1.21	1.19	1.18
.92	1.11	1.1	1.09	1.08	1.08	1.07
.91	.98	.98	.99	.99	.99	.99
.9	.88	.89	.9	.91	.91	.92

SAMPLE SIZE = 30 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	76.73	51.72	37.23	28.19	22.21	18.06
.99	7.64	6.35	5.45	4.78	4.28	3.88
.98	3.8	3.37	3.05	2.8	2.6	2.44
.97	2.52	2.32	2.17	2.04	1.94	1.86
.96	1.89	1.78	1.7	1.63	1.57	1.53
.95	1.5	1.45	1.4	1.37	1.34	1.31
.94	1.25	1.22	1.2	1.19	1.17	1.16
.93	1.06	1.06	1.05	1.05	1.05	1.04
.92	.93	.93	.94	.94	.95	.95
.91	.82	.83	.85	.86	.87	.88
.9	.73	.75	.77	.79	.8	.81

SAMPLE SIZE = 35 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	65.77	44.95	32.74	25.03	19.89	16.3
.99	6.55	5.52	4.79	4.25	3.83	3.5
.98	3.26	2.93	2.68	2.48	2.33	2.2
.97	2.16	2.02	1.9	1.81	1.74	1.68
.96	1.62	1.55	1.49	1.45	1.41	1.38
.95	1.29	1.26	1.24	1.22	1.2	1.19
.94	1.07	1.06	1.06	1.05	1.05	1.05
.93	.91	.92	.93	.93	.94	.94
.92	.79	.81	.83	.84	.85	.86
.91	.7	.73	.75	.76	.78	.79
.9	.63	.66	.68	.7	.72	.74

SAMPLE SIZE = 40 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	57.55	39.82	29.29	22.59	18.08	14.91
.99	5.73	4.89	4.29	3.83	3.48	3.21
.98	2.85	2.6	2.4	2.24	2.12	2.01
.97	1.89	1.79	1.7	1.64	1.58	1.53
.96	1.42	1.37	1.34	1.31	1.28	1.26
.95	1.13	1.12	1.11	1.1	1.09	1.08
.94	.94	.94	.95	.95	.95	.96
.93	.8	.82	.83	.84	.85	.86
.92	.7	.72	.74	.76	.77	.79
.91	.62	.64	.67	.69	.71	.72
.9	.55	.58	.61	.63	.65	.67

SAMPLE SIZE = 45 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	51.16	35.77	26.55	20.64	16.62	13.78
.99	5.1	4.4	3.89	3.5	3.2	2.96
.98	2.54	2.33	2.17	2.05	1.95	1.86
.97	1.68	1.61	1.55	1.5	1.45	1.42
.96	1.26	1.23	1.21	1.19	1.18	1.17
.95	1	1	1	1	1	1
.94	.83	.85	.86	.87	.88	.89
.93	.71	.73	.75	.77	.78	.8
.92	.62	.65	.67	.69	.71	.73
.91	.55	.58	.61	.63	.65	.67
.9	.49	.52	.55	.58	.6	.62

SAMPLE SIZE = 50 CONSUMER RISK = .1

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	46.04	32.51	24.32	19.03	15.42	12.85
.99	4.59	3.99	3.56	3.23	2.97	2.76
.98	2.28	2.12	1.99	1.89	1.81	1.74
.97	1.52	1.46	1.42	1.38	1.35	1.32
.96	1.13	1.12	1.11	1.1	1.09	1.09
.95	.9	.91	.92	.93	.93	.94
.94	.75	.77	.79	.8	.81	.83
.93	.64	.67	.69	.71	.73	.74
.92	.56	.59	.61	.64	.66	.68
.91	.49	.53	.56	.58	.6	.63
.9	.44	.48	.51	.53	.56	.58

SAMPLE SIZE = 10 CONSUMER RISK = .2

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	160.89	101.38	68.99	49.82	37.68	29.59
.99	16.02	12.45	10.09	8.45	7.26	6.36
.98	7.97	6.6	5.64	4.94	4.41	3.99
.97	5.29	4.55	4.01	3.6	3.29	3.04
.96	3.95	3.49	3.14	2.88	2.67	2.5
.95	3.14	2.83	2.6	2.41	2.27	2.15
.94	2.61	2.39	2.22	2.09	1.98	1.9
.93	2.22	2.07	1.95	1.85	1.77	1.71
.92	1.94	1.82	1.73	1.66	1.6	1.56
.91	1.71	1.63	1.57	1.51	1.47	1.43
.9	1.53	1.47	1.43	1.39	1.36	1.33

SAMPLE SIZE = 15 CONSUMER RISK = .2

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	107.26	70.13	49.21	36.47	28.21	22.58
.99	10.68	8.61	7.2	6.19	5.43	4.85
.98	5.32	4.57	4.03	3.62	3.3	3.05
.97	3.53	3.15	2.86	2.64	2.46	2.32
.96	2.63	2.41	2.24	2.11	2	1.91
.95	2.1	1.96	1.85	1.77	1.7	1.64
.94	1.74	1.65	1.59	1.53	1.49	1.45
.93	1.48	1.43	1.39	1.36	1.33	1.3
.92	1.29	1.26	1.24	1.22	1.2	1.19
.91	1.14	1.13	1.12	1.11	1.1	1.09
.9	1.02	1.02	1.02	1.02	1.02	1.02

SAMPLE SIZE = 20 CONSUMER RISK = .2

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	80.45	53.99	38.72	29.23	22.97	18.64
.99	8.01	6.63	5.67	4.96	4.42	4.01
.98	3.99	3.52	3.17	2.9	2.69	2.52
.97	2.65	2.42	2.25	2.12	2.01	1.92
.96	1.98	1.86	1.77	1.69	1.63	1.58
.95	1.57	1.51	1.46	1.42	1.38	1.36
.94	1.31	1.27	1.25	1.23	1.21	1.2
.93	1.11	1.1	1.09	1.09	1.08	1.08
.92	.97	.97	.98	.98	.98	.98
.91	.86	.87	.88	.89	.9	.9
.9	.77	.79	.8	.82	.83	.84

SAMPLE SIZE = 25 CONSUMER RISK = .2

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	64.36	44.08	32.15	24.62	19.59	16.06
.99	6.41	5.42	4.71	4.18	3.77	3.45
.98	3.19	2.87	2.63	2.44	2.29	2.17
.97	2.12	1.98	1.87	1.78	1.71	1.65
.96	1.58	1.52	1.47	1.42	1.39	1.36
.95	1.26	1.23	1.21	1.2	1.18	1.17
.94	1.05	1.04	1.04	1.04	1.03	1.03
.93	.89	.9	.91	.92	.92	.93
.92	.78	.8	.81	.82	.84	.85
.91	.69	.71	.73	.75	.77	.78
.9	.62	.64	.67	.69	.71	.73

SAMPLE SIZE = 30 CONSUMER RISK = .2

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	53.63	37.35	27.62	21.4	17.2	14.23
.99	5.34	4.59	4.04	3.63	3.31	3.06
.98	2.66	2.43	2.26	2.12	2.01	1.92
.97	1.77	1.68	1.61	1.55	1.5	1.46
.96	1.32	1.29	1.26	1.24	1.22	1.2
.95	1.05	1.05	1.04	1.04	1.04	1.04
.94	.87	.88	.89	.9	.91	.91
.93	.74	.76	.78	.8	.81	.82
.92	.65	.67	.7	.72	.73	.75
.91	.57	.6	.63	.65	.67	.69
.9	.51	.55	.57	.6	.62	.64

SAMPLE SIZE = 35 CONSUMER RISK = .2

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	45.97	32.46	24.29	19.01	15.4	12.84
.99	4.58	3.99	3.56	3.23	2.97	2.76
.98	2.28	2.12	1.99	1.89	1.8	1.74
.97	1.51	1.46	1.41	1.38	1.35	1.32
.96	1.13	1.12	1.11	1.1	1.09	1.09
.95	.9	.91	.92	.92	.93	.93
.94	.75	.77	.79	.8	.81	.83
.93	.64	.67	.69	.71	.73	.74
.92	.56	.59	.61	.64	.66	.68
.91	.49	.53	.55	.58	.6	.62
.9	.44	.48	.51	.53	.56	.58

SAMPLE SIZE = 40 CONSUMER RISK = .2

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	40.23	28.75	21.73	17.15	14	11.74
.99	4.01	3.53	3.18	2.91	2.7	2.53
.98	2	1.88	1.78	1.7	1.64	1.59
.97	1.33	1.29	1.27	1.24	1.22	1.21
.96	.99	.99	.99	.99	.99	1
.95	.79	.81	.82	.83	.85	.86
.94	.66	.68	.7	.72	.74	.76
.93	.56	.59	.62	.64	.66	.68
.92	.49	.52	.55	.58	.6	.62
.91	.43	.47	.5	.52	.55	.57
.9	.39	.42	.45	.48	.51	.53

SAMPLE SIZE = 45 CONSUMER RISK = .2

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	35.76	25.83	19.7	15.67	12.87	10.86
.99	3.56	3.18	2.89	2.66	2.48	2.34
.98	1.78	1.69	1.61	1.56	1.51	1.47
.97	1.18	1.16	1.15	1.14	1.13	1.12
.96	.88	.89	.9	.91	.91	.92
.95	.7	.73	.75	.76	.78	.79
.94	.58	.61	.64	.66	.68	.7
.93	.5	.53	.56	.59	.61	.63
.92	.43	.47	.5	.53	.55	.57
.91	.38	.42	.45	.48	.51	.53
.9	.34	.38	.41	.44	.47	.49

SAMPLE SIZE = 50 CONSUMER RISK = .2

R \ M =	1.0	1.1	1.2	1.3	1.4	1.5
.999	32.18	23.47	18.05	14.45	11.94	10.12
.99	3.21	2.89	2.64	2.45	2.3	2.18
.98	1.6	1.53	1.48	1.44	1.4	1.37
.97	1.06	1.06	1.05	1.05	1.05	1.04
.96	.79	.81	.83	.84	.85	.86
.95	.63	.66	.68	.7	.72	.74
.94	.53	.56	.59	.61	.63	.65
.93	.45	.48	.51	.54	.56	.59
.92	.39	.43	.46	.49	.51	.54
.91	.35	.38	.41	.44	.47	.49
.9	.31	.35	.38	.41	.43	.46

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